

Vibration attenuation using a 3D cubic re-entrant metamaterial structure with tunable bandgaps induced by variations in the re-entrant angle

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Abstract. This study explores the vibration attenuation capabilities of three-dimensional metamaterial periodic structures, leveraging the properties of auxetic structures and phononic crystals in forming bandgaps. While most conventional periodic structures exhibit vibration attenuation within limited, fixed frequency ranges, achieving tunable bandgaps remains a significant challenge. In this work, a novel three-dimensional auxetic re-entrant cubic periodic structure is proposed. The bandgap characteristics of this structure were analysed with and without internal resonators. Furthermore, the bandgap was examined with respect to the re-entrant angle (θ), revealing that both the position and width of the phononic bandgap can be adjusted by modifying the re-entrant angle. Our findings show that as the re-entrant angle is increased, the frequency at which the stopbands start decreases. To achieve real-time bandgap tunability, external compression deformation was applied, causing the unit cell to compress in both directions. This compression alters the angle of the re-entrant structure, which in turn changes the frequencies and modes of the bandgap, demonstrating a viable pathway to programmable control over elastic wave behaviour. The 3D auxetic structure can be utilised as a programmable vibration isolation device for a number of purposes, including wave filtering, waveguiding, and micro-vibration attenuation in precision engineering and aerospace systems.

Keywords: Re-entrant structure / auxetic metamaterial / periodic lattice / bandgap / vibration attenuation

1 Introduction

Vibration mitigation or isolation is crucial across a vast array of engineering disciplines—whether it's stabilizing heavy industrial machinery, enhancing vehicle performance, safeguarding civil structures, or ensuring the precision of instruments aboard spacecraft. Traditionally, passive vibration isolation methods primarily rely on springs and dampers to adjust the resonant frequency and reduce vibration transmission [1,2]. High-performance dampers, such as viscous fluid [3–5] or visco-elastic composite dampers [6,7], are a crucial part of the isolator design in passive systems to form the absorber structures to dissipate vibration energy [8]. However, these designs and

structures have many limitations such as low performance at low frequencies, temperature and strain dependency [9] etc., prompting research to create alternative passive vibration isolators based on quasi-zero-stiffness mechanism [10–14] as well as metamaterials [15–17].

Recent advancements have demonstrated the potential of mechanical metamaterials and phononic crystals for vibration isolation and control due to their exceptional ability to attenuate vibrations across a broad frequency range [18]. Just as photonic crystals control the propagation of light waves and phononic crystals control the propagation of sound waves, elastic waves can also be manipulated by structuring materials in a periodic manner to create band gaps and control wave propagation [19–23]. This proves particularly useful in engineering applications where the control of mechanical vibrations or seismic waves is important, such as in vibration isolation [17,24,25], wave

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guiding [26], or the development of new materials with tailored multi-functional properties [27,28]. These metamaterials and phononic crystals are typically formed by arranging materials with different acoustic or elastic properties in a periodic pattern, such as alternating layers of materials with varying densities or elastic properties. By precisely controlling the geometry [22] and composition of these structures [21], phononic crystals can exhibit interesting and useful elastic wave phenomena, such as the ability to manipulate the direction, speed, and attenuation of elastic waves by blocking certain frequencies or wavelengths from propagation through the material (elastic band gaps), or guiding sound waves along specific pathways.

On the other hand, auxetic metamaterials present very interesting features such as improved mechanical properties originating from negative Poisson's ratio, such as shear or indentation resistance, energy absorption capacity, fracture toughness, and the ability to control elastic wave propagation [29–33]. It is therefore very interesting to study the combination of such properties to obtain a metamaterial endowed with controllable phononic bandgaps to be tailored or enhanced during its functioning [19]. Moreover, the innovative additive manufacturing (3D-printing) processes, have provided the unprecedented manufacturing freedom for the fabrication of intricate parts and geometries [34].

Recently, auxetic metamaterial lattices especially those with re-entrant geometric construction have received widespread interest because of their improved mechanical properties and tuneable auxeticity compared to other conventional honeycomb structures [35]. Chen et al. [36] explored the interplay between Poisson's ratios and the elastic wave propagation occurring within an auxetic lattice metamaterials and demonstrated Bandgap tunability for vibration attenuation using an external mechanical stimulus. D'Alessandro et al. [33] propose a 3D simple cubic, single-phase Phononic crystal structure endowed with ultra-wide complete bandgaps whose tunability could be achieved by exploiting the negative Poisson's ratio of its unit cells. Tao et al. [37] designed an acoustic metamaterial plate based on the negative Poisson's ratio structure (NP-AMP) which achieved lower frequency and wider bandgap compared with traditional positive Poisson's ratio structures, and demonstrated that the bandgap range can be manipulated by the auxetic behaviour. In a study by Fei et al. [38], the bandgap tunability of a 3D anti-chiral auxetic metamaterial subjected to quasi-static loading was investigated. The results show that by applying rational tensile or compression force, the Poisson's ratio of the designed metamaterial can be changed from 0.45 to 0.35 with different cell parameters which in turn widened or narrowed the bandgap. Wei et al. [39] employ shape memory polymer (SMP) with special thermomechanical property to design a chiral auxetic metamaterial with tuneable band gap function which could be adjusted in real time by external stimuli (mechanical loadings, temperature field). Kheybari et al. [40] propose an anisotropic auxetic metamaterial that can attenuate both elastic vibrations and airborne sound waves simultaneously and in all directions and demonstrated that the attenuation

frequency ranges (Bandgaps) for both acoustic and elastic waves can be tuned by applying an external quasi-static load to the metamaterial structure. Qi et al. [41] presented a re-entrant and anti-chiral hybrid meta structures realized through rational distributions of rubber coated mass for wave attenuation. The study investigated the relationship between the geometrical parameters, local resonant inclusions schemes and wave attenuation performances of the proposed re-entrant and anti-chiral hybrid metamaterials.

However, in the studies highlighted, the periodic structure is either 2D or 3D with limited or no-load bearing capacity. In addition, the designed periodic structures can only have an attenuation within a fixed frequency range and adjustment of the bandgap has become a concern as seen in the studies of Elmadhi et al. [17] and An et al. [23]. Motivated by these results, this research investigates the attenuation performance of a novel 3D auxetic re-entrant cubic periodic structure produced using additive manufacturing (AM) processes. The cubic unit cell's design was influenced by its enhanced stability, greater stiffness and strength (structural rigidity), easy tunability as well as the ability to incorporate internal resonators. The vibration attenuation of the novel 3D auxetic re-entrant cubic periodic structure and the resulting effect of adding resonators on the attenuation performance of the structure was studied. In addition, the tunability of the bandgap frequencies by exploiting its auxetic behaviour through external mechanical stimulus, in this case uniaxial compression deformation was investigated. By using uniaxial compression, this phenomenon suggests that the new 3D auxetic re-entrant structure can be configured to suppress vibration over a wide frequency range. Thus, when the auxeticity and bandgap characteristics are properly combined, a 3D single-phase periodic structure with tunability of elastic wave attenuation can be realized. The 3D auxetic structure can be used as a programmable vibration isolation device in various applications such as micro vibration attenuation in aerospace and precision engineering, as well as wave filtering and wave guiding. The work is organized as follows: the mechanical design of the 3D auxetic re-entrant lattice, wave propagation and the bandgap formation mechanism are discussed in [Section 2](#). [Section 3](#) discusses the finite element analysis while [Section 4](#) highlights the experimental procedure. The results highlighting on effect of adding resonators and tunability of the bandgap by systematic parameter analysis is discussed in [Section 5](#), while conclusions are made in [Section 6](#).

2 Design and material

2.1 Geometry of the unit cell

The proposed novel 3D re-entrant lattice structure is consisting of unit cell made up of interconnected 2D re-entrant structures as depicted in [Figure 1a](#), compared with the conventional re-entrant 2D structure in [Figure 1b](#), alongside with the auxetic metamaterials lattice generated from tessellating the unit cell periodically in 3 dimensions as shown in [Figure 1c](#). The proposed structure of the unit cell can be completely defined by the following parameters:

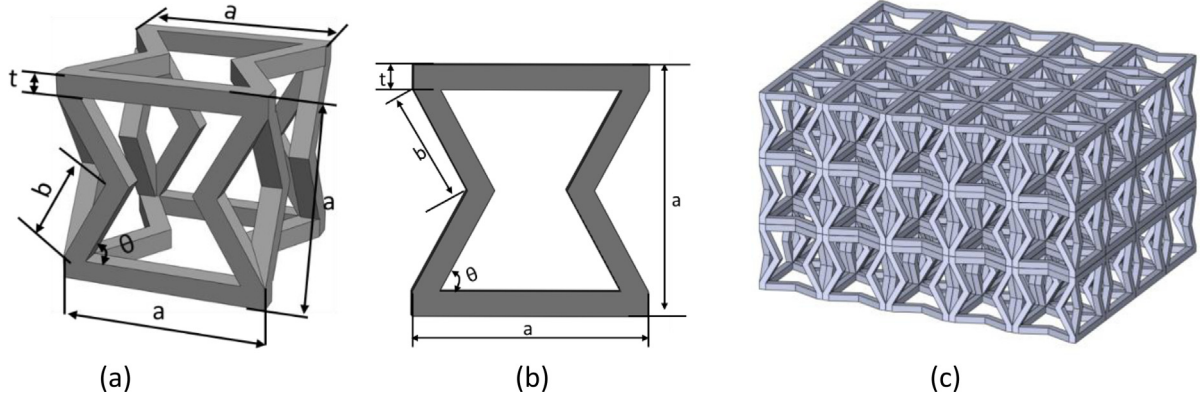


Fig. 1. 3D re-entrant auxetic structure: (a) structure of the unit cell, (b) 2D conventional re-entrant structure, (c) 3-dimensional periodic structure.

the side length of the cubic unit cell a , the angle of the inclined strut θ , the length of the inclined strut b , and the thickness of the strut t . The length of the inclined strut b is given by

$$b = \frac{(a - 2t)}{2 \sin \theta}. \quad (1)$$

The metamaterial lattice structure is generated by tessellating the unit cell periodically in 3D in x , y and z directions. The unit cell was modelled with a side length $a = 20$ mm, re-entrant angle $\theta = 60^\circ$, and the strut thickness $t = 2$ mm. When an external compressive force is applied on the two sides of the unit cell, it deforms in both directions with negative Poisson's ratio referred to as auxetic behaviour.

2.2 Bandgap enhancement by adding internal resonators

This section presents the consequences of adding local resonators for the variations of the bandgap of the 3D periodic structure. In metamaterials, bandgaps are often formed by Bragg scattering or local resonance, and the properties of the bandgap zones produced by these two methods differ greatly [42]. The addition of resonators alters the dispersion relations of the phononic crystal, which describes how wave frequency relates to wave vector. This alteration can lead to the emergence of new bandgaps or modify existing ones [43,44]. Using the simplified spring mass model, the 3D cubic structure was analysed as a periodic structure with a finite number of unit cells. The added internal resonators are composed of two struts, each with a diameter of d , connected to one side of a rectangular mass that is attached to the cubic lattice and has dimensions of $e \times f \times g$, as shown in Figure 2b.

Assuming all the resonators have an equivalent mass m_2 and stiffness k_2 , while the cubic structure has mass m_1 and stiffness k_1 , as depicted in Figure 2b, then the free vibration equation (no damping) of the j th cubic unit cell can be expressed [42] as

$$m_1 \ddot{x}_1(j) + k_1(2x_1^j - x_1^{j+1} - x_1^{j-1}) + k_2(x_1^j - x_2^j) = 0 \quad (2)$$

$$m_2 \ddot{x}_2(j) + k_2(x_2^j - x_1^j) = 0. \quad (3)$$

Re-writing in Matrix form

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1^j & \ddot{x}_2^j \end{bmatrix} + \begin{bmatrix} 2k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} x_1^j \\ x_2^j \end{bmatrix} - \begin{bmatrix} k_1(x_1^{j+1} + x_1^{j-1}) & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (4)$$

The harmonic solution for displacement of the j th unit cell is given based on Bloch theorem as

$$\begin{aligned} X^j &= Ae^{i(jqa - \omega t)} \\ X^{j+1} &= Ae^{i(jqa - \omega t)} e^{iqa} \\ X^{j-1} &= Ae^{i(jqa - \omega t)} e^{-iqa} \end{aligned} \quad (5)$$

where

a = length of the unit cell
 q = wave number
 ω = angular frequency
 A = displacement amplitude
 i = imaginary unit

$$[k(q) - M\omega^2]u = 0 \quad (6)$$

Then,

$$M_{eff} = m_1 + \frac{m_2 \omega_o^2}{\omega_o^2 - \omega^2} \quad (7)$$

Thus $\omega_o^2 = \frac{k_2}{m_2}$ when $\omega > \omega_o$, $m_{eff} < 0$, (i.e negative), we can also see that m_{eff} depends on both masses m_1 and m_2 and the frequencies ω and ω_o (mass and frequency).

From the lump mass, since the unit cell is a homogenous material, the effective stiffness k_{eff} is given as

$$k_{eff} = k_1 + \frac{1}{4}k_2 - \frac{1}{4} \left(m_1 \omega^2 + \frac{k_2 \omega_o^2}{\omega_o^2 - \omega^2} \right) \quad (8)$$

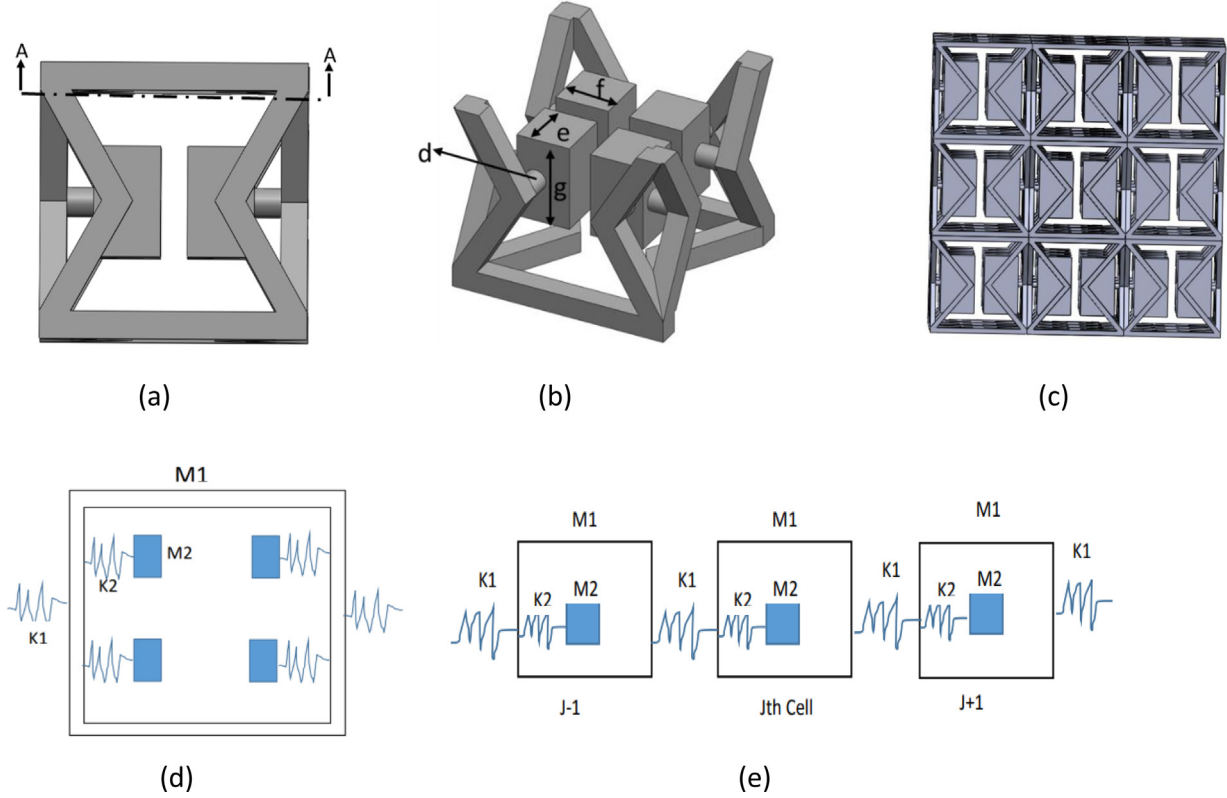


Fig. 2. 3D cubic auxetic structure with internal resonators: (a) cubic unit cell topology, (b) section view of the unit cell, (c) periodic lattice structure. (d) Schematic diagram of the simplified spring-mass model of the cubic re-entrant unit cell with external mass m_1 , internal resonators mass m_2 , external axial stiffness k_1 and internal axial stiffness k_2 . (e) Schematic diagram of the periodic lattice with a number of unit cells.

By taking equation (5), and considering second derivatives, This will give we arrive at

$$\begin{aligned} \ddot{X}^j &= -\omega^2 A e^{i(jqa - \omega t)} = -\omega^2 x^j \ddot{X}^{j+1} \\ &= -\omega^2 A e^{i(jqa - \omega t)} e^{iqa} = -\omega^2 x^{(j+1)} e^{iqa} \ddot{X}^{j-1} \\ &= -\omega^2 A e^{i(jqa - \omega t)} e^{-jqa} = -\omega^2 x^{(j-1)} e^{-iqa} \end{aligned} \quad (9)$$

$$-m_1 \omega^2 x_1^j + 2k_1 (1 - \cos(qa)) x_1^j + k_2 \left(1 - \frac{k_2}{k_2 - m_2 \omega^2}\right) x_1^j = 0 \quad (12)$$

Substituting (9) into (4),

$$\begin{bmatrix} -\omega^2 m_1 & 0 \\ 0 & -\omega^2 m_2 \end{bmatrix} \begin{bmatrix} x_1^j \\ x_2^j \end{bmatrix} + \begin{bmatrix} 2k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} x_1^j \\ x_2^j \end{bmatrix} - \begin{bmatrix} k_1 (e^{iqa} + e^{-iqa}) x_1^j \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (10)$$

where $e^{iqa} + e^{-iqa} = 2 \cos(qa)$

$$\begin{bmatrix} -\omega^2 m_1 & 0 \\ 0 & -\omega^2 m_2 \end{bmatrix} \begin{bmatrix} x_1^j \\ x_2^j \end{bmatrix} + \begin{bmatrix} 2k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} x_1^j \\ x_2^j \end{bmatrix} - \begin{bmatrix} 2k_1 \cos(qa) x_1^j \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (11)$$

2.3 Bandgap tunability

The stiffness of the re-entrant structure is influence by its geometry including the re-entrant angle. Thus, the stiffness depends on how the structure deforms under load and can be modelled as the function of the re-entrant angle, young modulus, struts length and thickness as

$$k_1(\theta) = \frac{EA}{b(\theta)} \quad (14)$$

Substituting b from equation (1), we have

$$K_1(\theta) = \frac{2EA \sin \theta}{a - 2t}. \quad (15)$$

The bandgap frequencies are determined by the natural frequency of the unit cell, which depends on the effective stiffness and mass

$$f(\theta) = \frac{1}{2\pi} \sqrt{\frac{2EA \sin \theta}{m(a - 2t)}}. \quad (16)$$

3 Numerical simulations

In this study, the dynamic behaviour of the proposed periodic structure is described as a typical phononic crystal using dispersion analysis performed over the unit cell using Bloch's periodic boundary conditions. The software COMSOL 6.0 was used for the FE simulation. The model is solved by eigenfrequency analysis to calculate the dispersion curves and consequent bandgaps of the structure by applying Bloch-Floquet periodic conditions to all the edges of the unit cell. The frequency is normalized to a non-dimensional frequency (f_n) defined as the ratio between the product of the frequency f and the unit cell dimension a , and the sound velocity in the material given as $f_n = f_a/v$, and the sound velocity in the material expressed as the square root of the ratio of the young modulus and the density of the material.

The dispersion curves were constructed to explore the bandgap properties of the re-entrant 3D cubic structure where the normalized frequency (f_n) is plotted against the wave vector along the path of the irreducible Brillouin zone of the unit cell from Γ -X-M- Γ -R-X-M-R as shown in Figure 3. In addition, vibration modal analysis is conducted to understand the formation mechanisms of these bandgaps.

The bandgap property of the structure can be tuned by changing the values of t/a as well as θ . The accuracy of the numerical dispersion curves was verified by calculating the transmission loss spectrum of elastic waves in the vertical direction of a finite periodic structure made up of $5 \times 4 \times 3$ unit cells (Fig. 1c). Displacement excitation is applied to the bottom side of the structure along the vertical direction, and the displacement response is measured at the top. Periodic boundary conditions are imposed with two perfectly matched layers (PMLs) placed at both ends to prevent energy reflection and ensure the accuracy of the calculations [37]. The transmissibility ratio in terms of acceleration levels (T) is expressed as

$$T = 20 \log_{10} \frac{A_{out}}{A_{in}} \quad (17)$$

where, A_{out} and A_{in} are the amplitudes of the measured output and input acceleration signals, respectively.

The dispersion curves of unit cell with strut thickness $t=1.5$, unit cell size $a = 20$ mm and the re-entrant angle of 55 degrees alongside the responding frequency response spectrum indicates the presence of stopbands as shown in

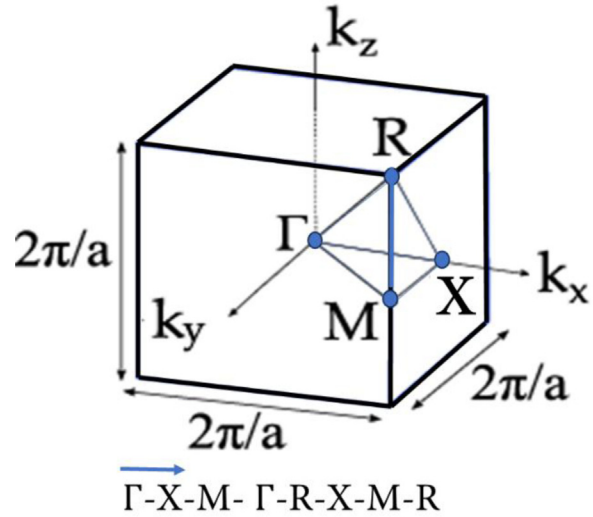


Fig. 3. The path of the wave vector along the Irreducible Brillouin zone of the unit cell.

Figure 4. It can be seen from the figure that a nearly complete bandgap occurs at low normalized frequency of 0.01, while additional two complete bandgaps are observed at normalised frequencies of 0.03 and 0.035. The transmission response spectrum of the periodic structure is basically consistent with the bandgap frequency in the corresponding dispersion diagram with minor deviations as shown in Figure 4b. The results show that the novel 3D cubic re-entrant structure can attenuate elastic waves at low frequency.

4 Experimental procedure

A representative sample of the 3D cubic lattice metamaterial's structure with unit cell dimensions of 10, 15, 20, and 30 mm was fabricated using an additive manufacturing process using PA12 polymer material. The material properties for PA12 can be found in Table 1. A dynamic experiment was conducted to demonstrate vibration transmissibility of the fabricated 3D auxetic metamaterial structure. An experimental setup was assembled, comprising a broadband vibration shaker, and dedicated signal generation and acquisition units. The samples were tested using the experimental setup presented in Figure 5. A mechanical vibration shaker was employed to excite the metamaterial lattice structure with an input signal over a frequency range of 20 Hz–5 kHz. The metamaterial samples were adhesively affixed on one side to a connector which was in turn bolted through a stinger to the mechanical shaker. An acceleration sensors were attached on the top and the bottom surfaces of the metamaterials structure to acquire the input and output signals respectively [45]. The metamaterial samples were then suspended using piano strings to approximate free-free boundary conditions, this approach of suspending the tested structures has been discussed and proved effective in various researches [17,19,46,47]. The acceleration sensors

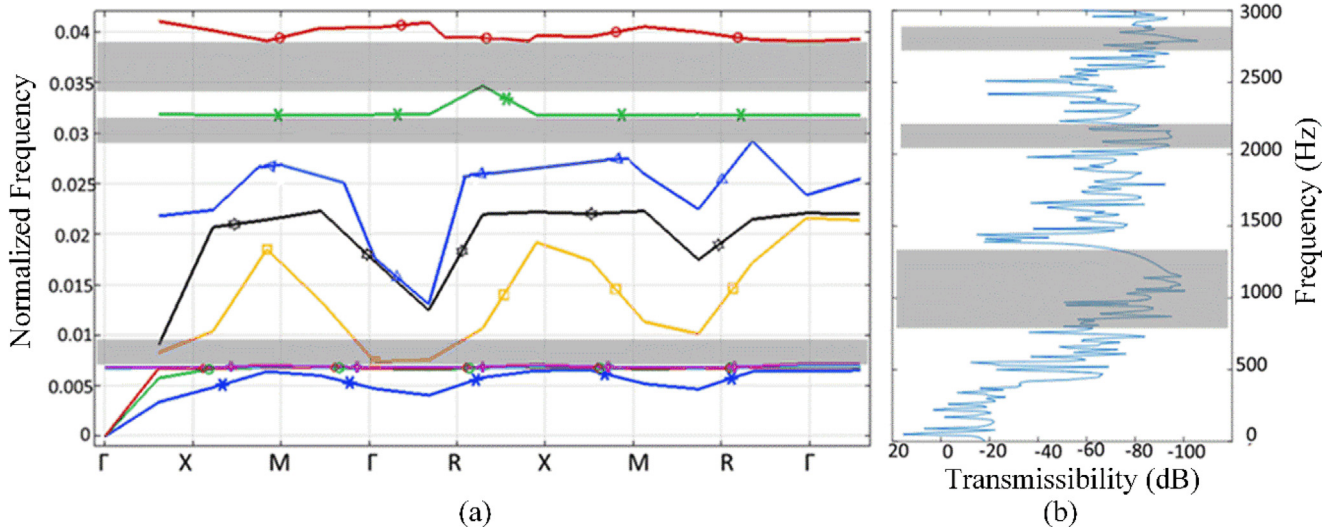


Fig. 4. Dispersion diagram and transmissibility of the novel 3D cubic re-entrant structure (a). Dispersion diagram of a unit cell with a strut thickness to unit cell size ratio (t/a) = 0.075 and 55 degrees re-entrant angle. (b) Vibration transmissibility for the 3D cubic lattice structure with $5 \times 4 \times 3$ unit cells configuration.

Table 1. Properties of the PLA 12 polymer.

Density	1030 kg/m ³
Elastic Modulus	1.935 E 6 kN/m ²
Poisson's ratio	0.34

were then connected to the signal acquisition module. A white noise signal was generated by a signal generator and then amplified by a power amplifier. The acceleration data within the tested frequency range were obtained through the acceleration sensor.

5 Results and discussion

5.1 Parametric studies

Figure 6 depicts the dispersion curves of multiple cubic re-entrant auxetic structures with different unit cell sizes (a) and constant $t=1.5$ mm, resulting in varying t/a ratios. Figures 6a–6d show the dispersion curves and bandgap of a unit cell with strut thickness to cell size ratios of 0.15, 0.1, 0.075, and 0.05, respectively. At 0.15 ratio, two narrow bandgaps with different bandwidths are visible at normalised frequencies of 0.32 and 0.41. In contrast, when the thickness of the strut to unit cell size ratio decreases to 0.1, 0.075, and 0.05 (Figs. 6c and 6d), we can observe a shift in the bandgap location to lower normalised frequencies as well as the widening of the start and stop of the bandgap. The ratio provides an indication of an increase in the solid volume fraction (though not directly). A higher ratio generally means a greater proportion of the unit cell is solid material, affecting the overall density and mechanical properties of the material. Larger unit cells (lower ratio of t/a) can support a greater number of modes. In comparison, smaller unit cells (higher ratio of t/a) may exhibit higher stiffness and density, influencing bandgap formation at higher frequencies

To further investigate the bandgap properties of the cubic re-entrant structure, the effects of unit cell size at constant strut thickness to unit cell size ratio t/a ($t/a = 0.1$ at 1/10, 1.5/15, 2/20, and 3/30) on the width and position of the bandgap were investigated and presented in Figures 7a–7d. From the figures, we can observe that as the unit cell size increases, one of the two narrow stopbands in the 10 mm unit cell at 0.11 and 0.12 enlarges and forming a single stopband at the normalized frequency of 0.12. The stopband continues to enlarge as the unit cell sizes increases from 10 mm through 30 mm. This demonstrates that Bragg scattering is the dominant mechanism for bandgap formation, as the lattice's periodicity, dictated by cell size, causes Bragg scattering of elastic waves. The size of the unit cell determines the effective wavelengths of elastic waves that can be manipulated. Smaller unit cells tend to create bandgaps at higher frequencies, while larger cells can facilitate bandgaps at lower frequencies [48,49]. The cell size affects how different elastic wave modes in longitudinal and transverse are couple with one another which in turn, affects the width and position of the bandgaps.

5.2 Effect of adding resonating mass on the bandgap

Compared with the dispersion diagrams of the auxetic cubic unit cell without internal resonators, Figure 8 presents the dispersion relation and the band structures of the unit cell with added internal resonators. In contrast, Figure 9 shows the mode shapes at the start and end of the second stopband. It is impressive to observe that adding resonating mass has a drastic impact on the frequency and depth of the bandgap width. It can be seen that the stopbands shift to lower frequencies and creating more bandgaps. Figure 8a corresponds to the dispersion diagram for the 10 mm unit cell which recorded two complete bandgaps that start at normalized frequency of 0.096 and 0.12 and end at 0.112 and 0.127 respectively. However,

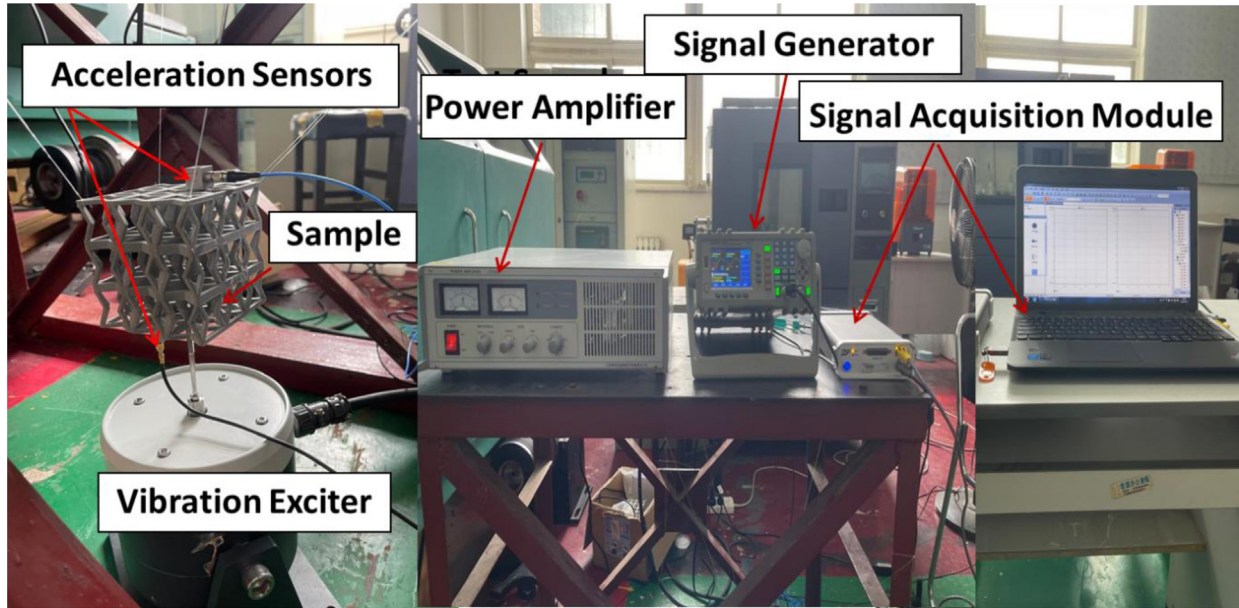


Fig. 5. Experimental platform to measure the vibration transmissibility.

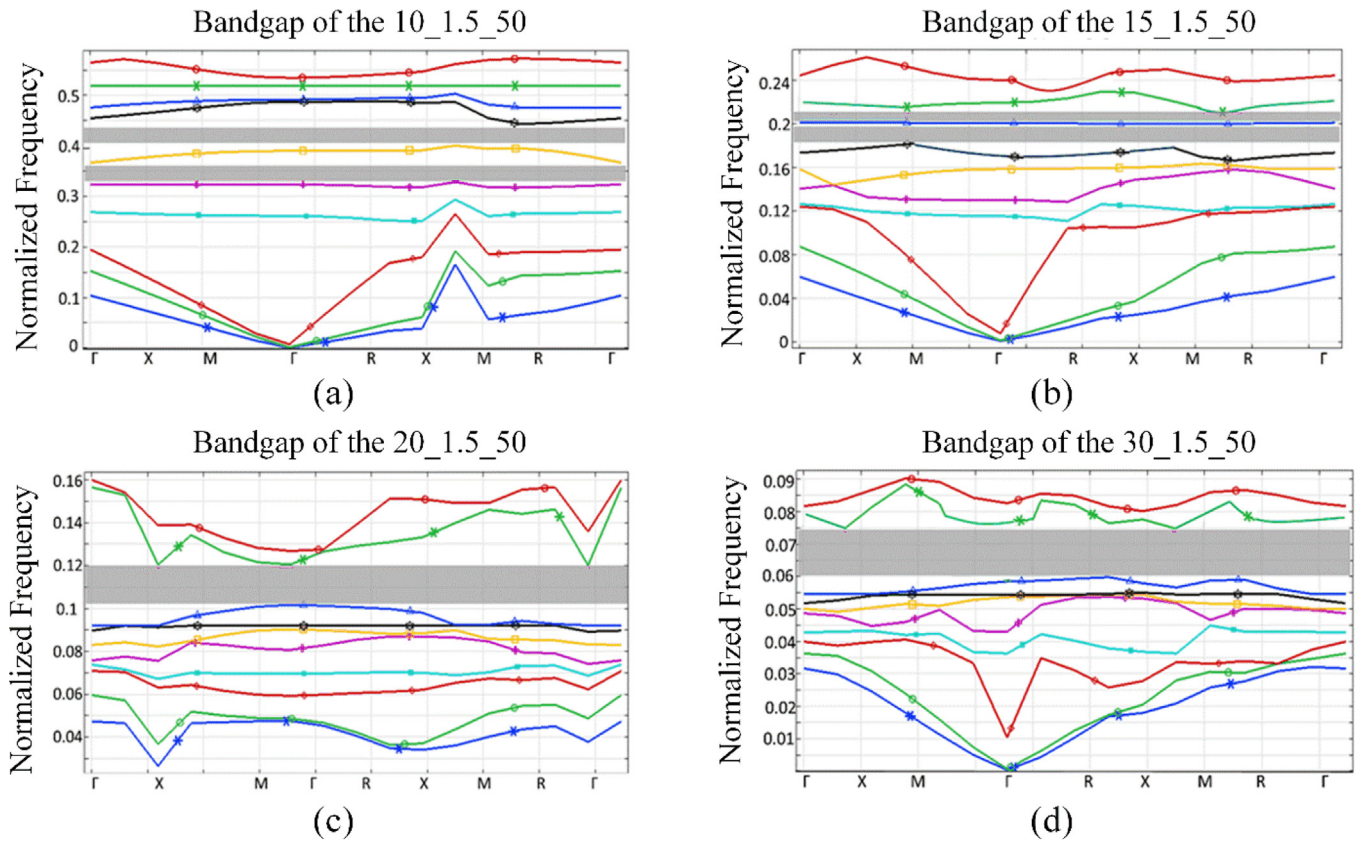


Fig. 6. Dispersion diagrams of unit cell with varying struts thicknesses to unit cell size ratio (t/a) and fixed re-entrant angle θ . Figures (a)–(d) correspond to the dispersion curves of $t/a=0.15$, 0.1, 0.075 and 0.05 respectively.

when unit cell size is increased, the first stopband frequency decreases to normalized frequency of 0.075 while the second stopband disappeared completely. The dispersion diagram for 20 mm and 30 mm unit cells indicates a further decrease in the stopband frequency with enhanced bandgaps in

terms of low frequency and bandwidth compared to the periodic lattice without the resonators. The local resonance of the mass blocks is responsible for the formation of the bandgaps. The vibration modes shapes at the start and end of the second stopband corresponding to strut thickness to

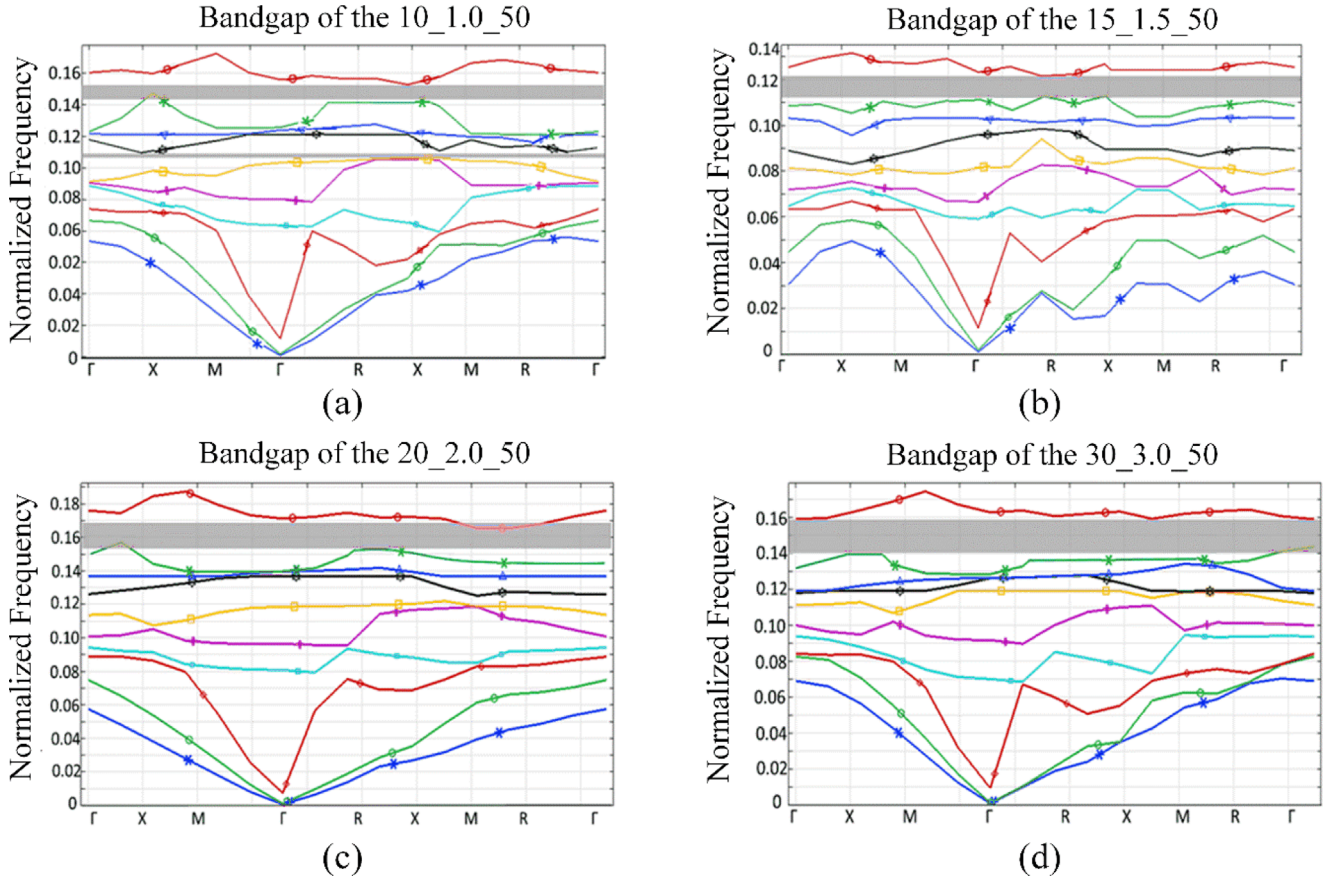


Fig. 7. Dispersion diagrams of unit cell with varying unit cell size and fixed struts thicknesses to unit cell size ratio of $t/a=0.1$ and fixed re-entrant angle θ . **Figures 7a–7d** correspond to the dispersion curves $a=10, 15, 20$ and 30 mm respectively.

unit cell size ratio (t/a)=0.05, 0.075, 0.1 and 0.15 respectively are shown in **Figure 9**. The modes mainly show the bending deformation of the beams and struts holding the internal resonators as well as the movement of the resonators.

The bandgap properties of the 3D cubic periodic lattice structures were verified by calculating the transmission loss spectrum of the elastic waves in the z -direction of the periodic structure made up $5 \times 4 \times 3$ unit cells and is experimentally tested by method describe in **Figure 5**. The transmission spectrums are presented in **Figure 10**. **Figure 10a** represents the transmission spectrum of unit cell size $a=10$ mm, it can be observed that there is no large attenuation in the transmission spectrum within the tested frequency as the first complete stopband occurred between normalized frequency 0.095 and 0.11. However, we can observe low attenuation at normalized frequency of 0.02 which is as a result of partial bandgap at that frequency region. **Figures 10b–10d** represent the transmission spectra for 15, 20 and 30 mm unit cell respectively. It can be observed that the start and end of the bandgap region of the experimental transmissibility response exhibits the same general trend as the simulation, with attenuation zones appearing in similar frequency ranges.

However, the measured band gaps are shallower and less distinct, and the resonance peaks are more damped compared to the sharp features predicted numerically. These discrepancies can be attributed to the non-ideal

boundary conditions that arise from approximating free-free boundary conditions, which involve suspending the structure with a piano string, as described in **Section 4** as observed in related studied [47,50]. Additionally, the shaker is connected to the sample via a stinger glued to its surface, which introduces additional compliance and modes; consequently, the force transmitted to the lattice is not uniform, as was assumed in the simulation. Moreover, the dynamics of the shaker and stinger assembly, along with the force-driven excitation used in the experiment, differ from the displacement-driven boundary conditions assumed in the simulation. This mismatch could lead to discrepancies in the measured transmissibility. Furthermore, damping in the real system is more significant and complicated due to factors such as material hysteresis, joint friction, and adhesive losses, compared to the simplified loss model employed in COMSOL [51]. This increased damping results in reduced peak amplitudes and partially fills the band gaps as can be observed in the **Figure 10**.

5.3 Bandgap tunability

To study the effect of the re-entrant angle (θ) and the struts thickness (t) on the evolution of the bandgaps, we choose the unit cell with the widest stopband, i.e., $a=20$ mm, $t=1$ mm, and $\theta=55$ degree (**Fig. 7c**), and examine the

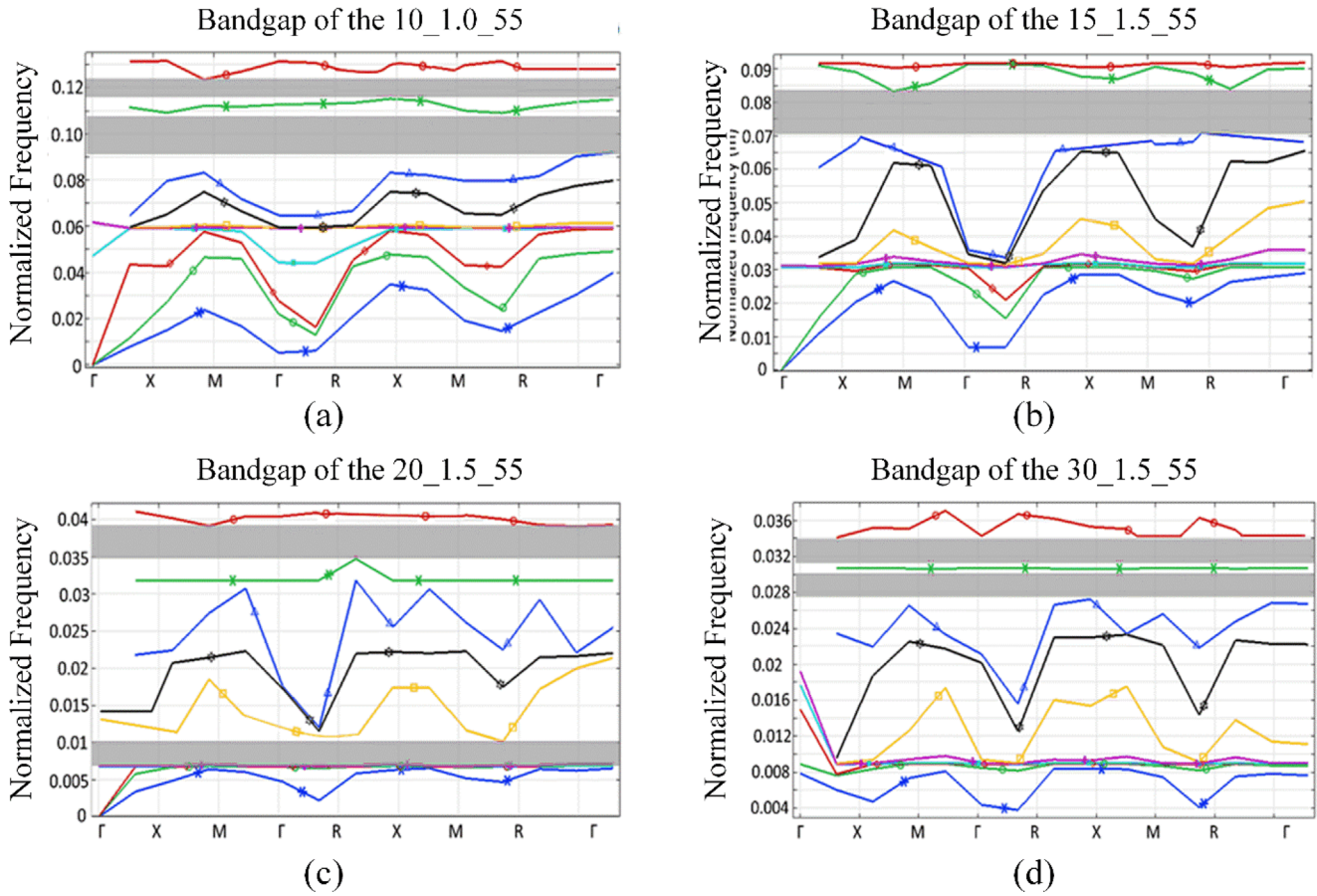


Fig. 8. Dispersion diagrams of unit cell with internal resonators. Figures 8a–8d correspond to the dispersion curves of unit cells $a=10$, 15, 20 and 30 mm.

bandgap tunability by manipulating the unit cell parameters (θ) and the struts thickness to unit cell size ratio (t/a). The re-entrant angle was varied from 50 degrees through 75 at the step of 5 degrees, while the t/a was varied from 0.05, 0.075, 0.1, and 0.15, i.e., $t=1$, 1.5, 2, and 3 mm, respectively, at a constant unit cell size of 20 mm. Figure 11 presents the influence of varying parameters on the start and end of the stopband frequencies on the novel 3D cubic re-entrant periodic structure. Figure 11a shows that increasing the re-entrant angle has less of an impact on the first stopband in terms of both position and bandgap width than it does on the second stopband. Additionally, we observed that when the re-entrant angle increases, the frequency at which the stopband starts decreases. However, there are no specific trends in terms of the width of the bandgap, with 55 degrees recording the widest bandgap while 60 degrees recording the lowest bandwidth. It can be concluded that designing the re-entrant cubic structure with larger angles will result in attenuation at lower frequencies compared to smaller re-entrant angles. Regarding the strut thickness to unit cell size ratio, Figure 11b shows that as the ratio increases, the bandgap position shifts to higher frequencies while also widening. As the thickness to unit cell size ratio t/a increases, the bandgap width enlarges where it is more pronounced in the second stopband while shifting to a higher frequency. This will enable a design of a tailored periodic

structures with target frequency of stopband and bandwidth by manipulating the design parameters of the unit cell.

The phononic bandgap position and width can be controlled by varying the re-entrant angle of the 3D auxetic cubic structure, as we have demonstrated. Thus, the novel structure can be designed to facilitate bandgap tunability by adjusting the re-entrant angle, allowing for real-time bandgap manipulation under small deformation. We examine the dynamic tunability of these bandgaps using an external mechanical stimulus, in this case uniaxial compression deformation at different compression strains, as it is evident that these bandgaps can be mechanically adjusted by altering the angle of the re-entrant cubic structure. In the proposed novel 3D cubic re-entrant periodic structure, the tunability of the bandgap is achieved due to the compression deformation in two orthogonal directions due to the auxeticity of the unit cell. The unit cell compresses in both directions when the compressive load is applied in the Z-direction. This compression alters the angle of the re-entrant structure, consequently changing the frequencies and modes of the bandgap through either a softening or hardening effect. Figures 12 and 13 depict the bandgap's fluctuation with varying strain values.

Figure 13 shows the continuous evolution of the bandgap as the compression strain increases for both the unit cells $a=20$ mm and 10 mm. Compared with the

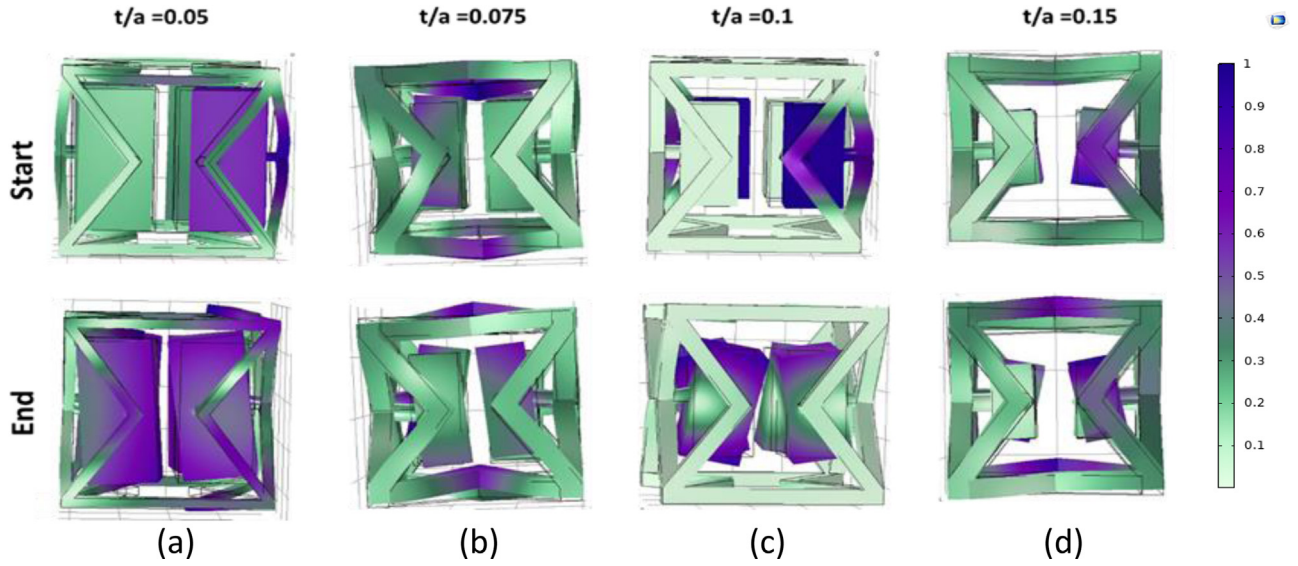


Fig. 9. Mode shapes at the start and end of the second stopband corresponding to strut thickness to unit cell size ratio (t/a)=0.05, 0.075, 0.1 and 0.15 respectively.

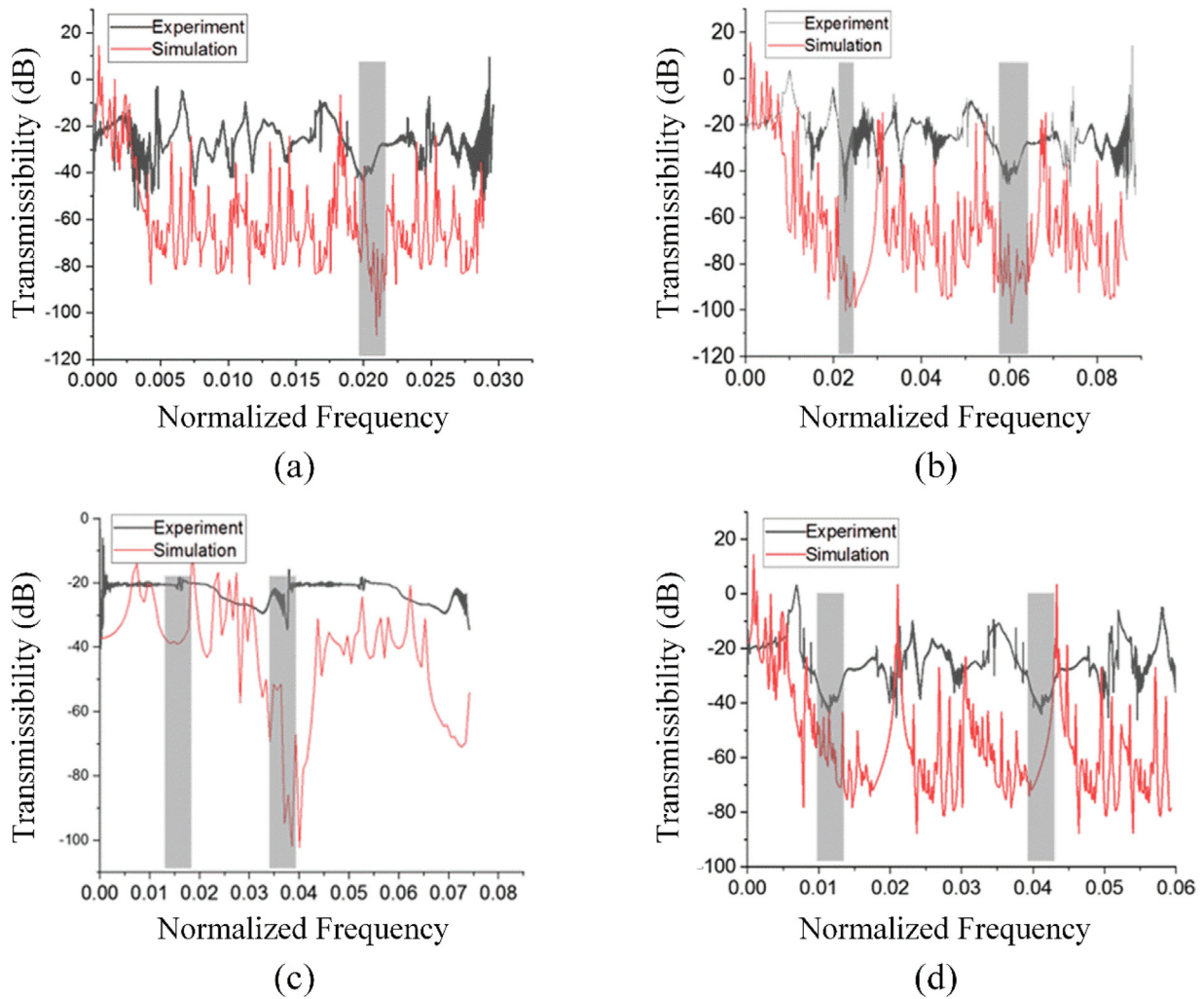


Fig. 10. Experimental results for vibration transmissibility acquired for the 3D cubic lattice structure with $5 \times 4 \times 3$ unit cells compared with the simulation results: (a) lattice structure with unit cell dimension $a=10$, $t/a=0.1$, (b) lattice structure with unit cell dimension $a=15$, $t/a=0.1$, (c) lattice structure with unit cell dimension $a=20$, $t/a=0.075$, (d) lattice structure with unit cell dimension $a=30$, $t/a=0.05$.

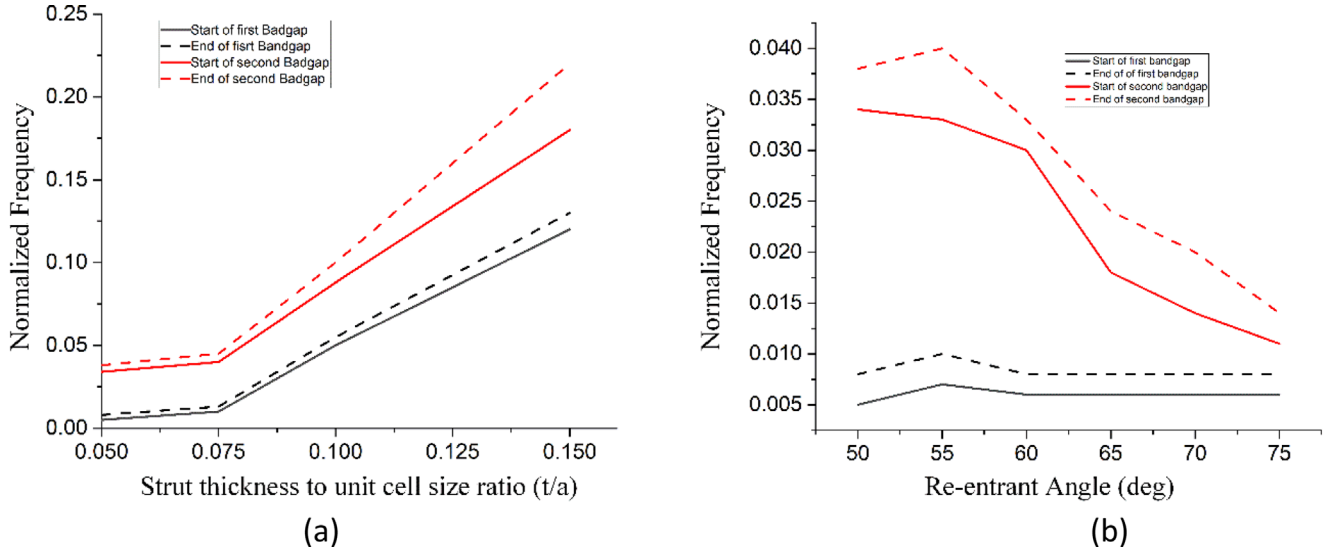


Fig. 11. Influence of the unit cell parameters on the bandgap of the periodic structure. (a) Effects of the re-entrant angle on the bandgap. (b) Effect of struts thickness to cell size ratio t/a on the evolution of the bandgap.

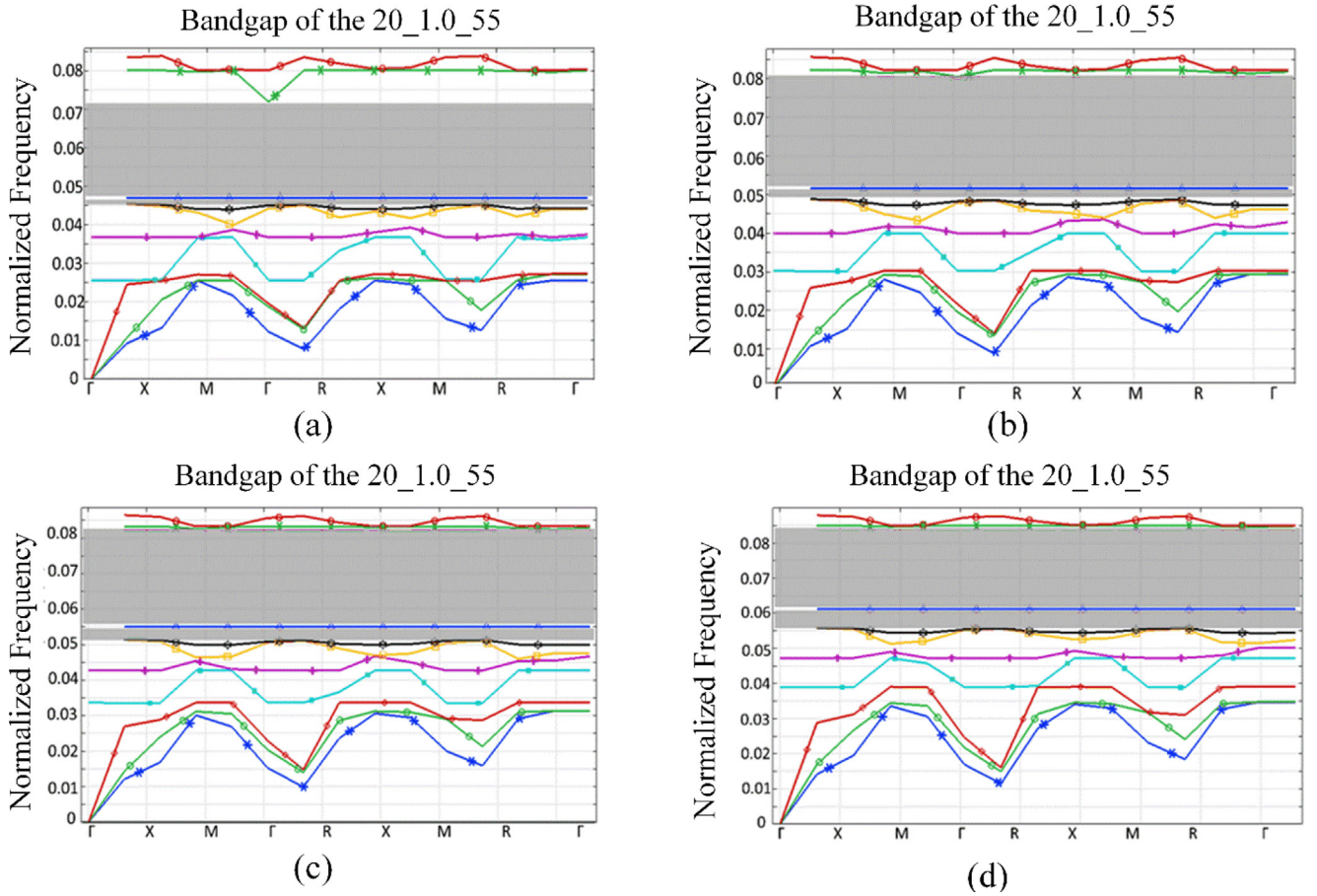


Fig. 12. Dispersion diagram of the unit cell $a=20$ mm, $t=1$ mm and $\theta=55^\circ$ under different compression strain: (a) – (d) dispersion diagram at compression deformation corresponding to the nominal strain $-0.01, -0.02, -0.03$ and -0.05 , respectively.

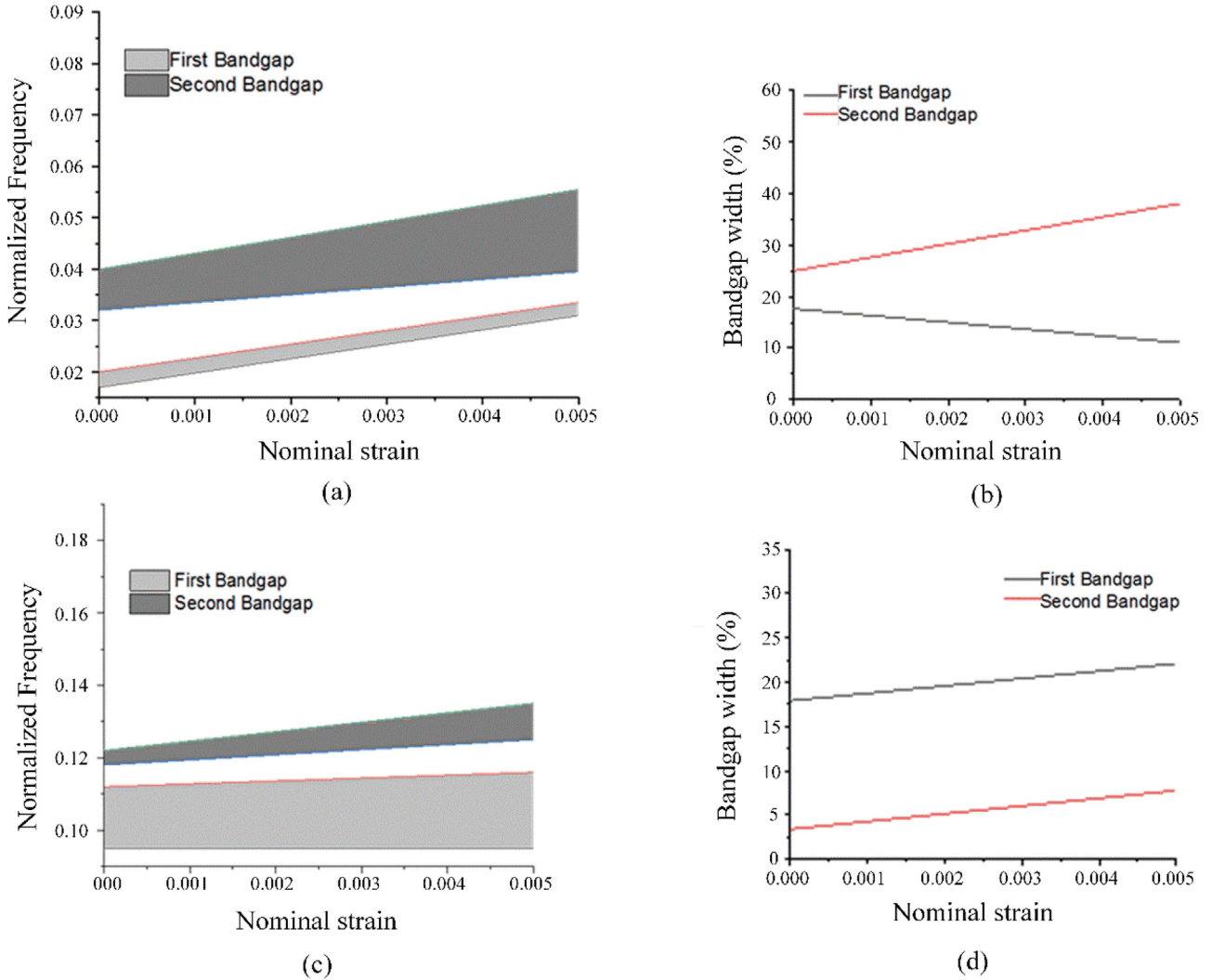


Fig. 13. Evolution of the bandgap with nominal strain increase from 0 to -0.05 . (a) Evolution of bandgap for 20 mm unit cell size. (b) Relative width of the first and second bandgaps for 20 mm unit cell. (c) Evolution of bandgap for 10 mm unit cell size. (d) Relative width of the first and second bandgaps for 10 mm unit cell.

non-deformed configurations, we can observe that both the first and second bandgap gradually move upwards as the compression increases. Figures 13b and 13d further present the relative width of the bandgaps. It can be observed that the wider bandgap in both cases evolves gradually widening as compression increases. However, the narrow bandgap in the 10 mm unit cell changes dramatically. As the compression strain is applied, the bandgap widens at -0.01 and shrink all through -0.03 and then widens at -0.04 . The above analysis indicates that although both changes in geometry and effective material properties can influence the bandgap formation in a structure, the both could be the main factors that influence the changes in the bandgap as the compression strain applied has significant effect on the geometry of the structure by changing its re-entrant angle.

6 Conclusion

In this work, we investigated the vibration attenuation performance of a novel 3D cubic re-entrant auxetic lattice

structure produced using an additive manufacturing process. The research demonstrates its significant potential for low-frequency vibration attenuation and its tunability to achieve a wide frequency bandgap. The research highlights how the strut thickness to unit cell size ratio (t/a) and the re-entrant angle (θ) affect the formation of bandgaps. It is noted that the bandgaps shift to lower frequencies and improve vibration attenuation with a lower ratio of the strut thickness to unit cell size (t/a) and a larger re-entrant angle. Significantly, the introduction of internal resonators has been shown to enhance bandgap characteristics, providing deeper and more numerous stopbands. Furthermore, the proposed novel 3D cubic re-entrant periodic structure exhibits tunability of the bandgap through compression deformation in two orthogonal directions due to the auxeticity of the unit cell. When the compressive load is applied in the Z-direction, the unit cell compresses in both directions. This compression alters the angle of the re-entrant structure, consequently changing the frequencies and modes of the bandgap.

The study confirms that the dynamic behaviour of the proposed periodic structures can be optimized for specific mechanical applications through systematic modifications in geometric parameters. The interaction between structural geometry and material properties will enable the creation of wide and robust bandgaps, which are primarily formed through a combination of Bragg scattering and local resonances. With valuable insights gained in this study in terms of manipulating the bandgap by changing the re-entrant angle, the tunability of these bandgaps through external stimuli–compression deformation in this case, further underscore the adaptability of these periodic structures for various engineering applications such as vibration control in aerospace to acoustic insulation in civil engineering. Further investigations on other methods of external stimulation such as temperature change, the scalability and long-term performance of these structures will be essential for their practical implementation in real-world scenarios.

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Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability statement

The data that support the findings in this work are included within the article. Additional data is available from the corresponding author upon reasonable request.

Author contribution statement

Musa Abubakar: Conceptualisation, software and writing the original draft. Huaitao Shi: Methodology, data curation and supervision. Xiaotian Bai: Supervision, funding acquisition and review. Tianzhi Yang: Supervision and Review. Ke Zhang: Review and editing. Xianming Sun: supervision, review and editing.

References

1. X. Feng, L. Yan, W. Chun, Review on vibration isolation technology, *J. Phys: Conf. Ser.* **1820**, 5–10 (2021) <https://doi.org/10.1088/1742-6596/1820/1/012009>
2. S. Kaul, Vibration isolation—background, in: *Modeling and Analysis of Passive Vibration Isolation Systems* (Elsevier, 2021), pp. 1–26 <https://doi.org/10.1016/B978-0-12-819420-1.00007-8>
3. Y. Zhao, J. Cui, X. Bian, L. Zou, S. Liang, L. Wang, Study on performance of an air spring isolator for large-scale precision optical micro-vibration isolation, *ACM Int. Conf. Proc. Ser. Part F16898*, 173–178 (2020) <https://doi.org/10.1145/3452940.3452974>
4. D.O. Lee, G. Park, J.H. Han, Experimental study on on-orbit and launch environment vibration isolation performance of a vibration isolator using bellows and viscous fluid, *Aerosp. Sci. Technol.* **45**, 1–9 (2015) <https://doi.org/10.1016/j.ast.2015.04.012>
5. G. Xu, Z. Gou, B. Zhang, Study of flexible spacecraft pointing control based on integrated vibration isolation and pointing stewart platform, in: *Signal and Information Processing, Networking and Computers. Lecture Notes in Electrical Engineering* (Springer, 2020), Vol. 628. https://doi.org/10.1007/978-981-15-4163-6_54
6. C. Deng, D. Mu, X. Jia, Z. Li, Effects of rubber shock absorber on the flywheel micro vibration in the satellite imaging system, *Photonic Sens.* **6**, 372–384 (2016) <https://doi.org/10.1007/s13320-016-0349-1>
7. M. Safarabadi, H. Izi, J. Haghsheenas, H.K. Kellardeh, Design of micro-vibration isolation system for a remote-sensing satellite payload using viscoelastic materials, *Eng. Solid Mech.* **8**, 69–76 (2020) <https://doi.org/10.5267/j.esm.2019.8.003>
8. Q. Wang et al., A metamaterial isolator with tunable low frequency stop-band based on magnetorheological elastomer and magnet spring, *Mech. Syst. Signal Process.* **208**, 111029 (2024) <https://doi.org/10.1016/j.ymssp.2023.111029>
9. M. Wang et al., Modeling and analysis of a four-parameter vibration isolator with frequency-dependent damping and its implementation based on GERF, *Smart Mater. Struct.* **32**, 075023 (2023) <https://doi.org/10.1088/1361-665X/acde24>
10. J. Tang, Y. Yang, Y. Li, D. Cao, A 6-DOF micro-vibration isolation platform based on the quasi-zero-stiffness isolator, *Proc. Inst. Mech. Eng. Part C J. Mech. Eng. Sci.* **235**, 6019–6035 (2021) <https://doi.org/10.1177/09544062211010831>
11. G. Yan, S. Wang, H.X. Zou, L.C. Zhao, Q.H. Gao, W.M. Zhang, Bio-inspired polygonal skeleton structure for vibration isolation: Design, modelling, and experiment, *Sci. China Technol. Sci.* **63**, 2617–2630 (2020) <https://doi.org/10.1007/s11431-020-1568-8>
12. X. Zhou, X. Sun, D. Zhao, X. Yang, K. Tang, The design and analysis of a novel passive quasi-zero stiffness vibration isolator, *J. Vib. Eng. Technol.* **9**, 225–245 (2021) <https://doi.org/10.1007/s42417-020-00221-6>
13. A. Zolfagharian, M. Bodaghi, R. Hamzehei, L. Parr, M. Fard, B.F. Rolfe, 3D-printed programmable mechanical metamaterials for vibration isolation and buckling control, *Sustainability* **14**, 6831 (2022). <https://doi.org/10.3390/su14116831>
14. Q. Zhang, D. Guo, G. Hu, Tailored mechanical metamaterials with programmable quasi-zero stiffness features for full-band vibration isolation, *Adv. Funct. Mater.* **31**, 2101428 (2021). <https://doi.org/10.1002/adfm.202101428>
15. M. Al Rifaie, H. Abdulhadi, A. Mian, Advances in mechanical metamaterials for vibration isolation: a review, *Adv. Mech. Eng.* **14**, 1–20 (2022) <https://doi.org/10.1177/16878132221082872>

16. J.C. Ji, Q. Luo, K. Ye, Vibration control based metamaterials and origami structures: a state-of-the-art review, *Mech. Syst. Signal Process.* **161**, 107945 (2021) <https://doi.org/10.1016/j.ymssp.2021.107945>
17. W. Elmadih, D. Chronopoulos, W.P. Syam, I. Maskery, H. Meng, R.K. Leach, Three-dimensional resonating metamaterials for low-frequency vibration attenuation, *Sci. Rep.* **9**, 11503 (2019) <https://doi.org/10.1038/s41598-019-47644-0>
18. H.T. Shi, M. Abubakar, X.T. Bai, Z. Luo, Vibration isolation methods in spacecraft: a review of current techniques, *Adv. Sp. Res.* **73**, 3993–4023 (2024) <https://doi.org/10.1016/j.asr.2024.01.020>
19. H. Meng, et al. 3D rainbow phononic crystals for extended vibration attenuation bands, *Sci. Rep.* **10**, 18989 (2020) <https://doi.org/10.1038/s41598-020-75977-8>
20. A.O. Krushynska et al., Accordion-like metamaterials with tunable ultra- wide low-frequency band gaps Accordion-like metamaterials with tunable ultra-wide low-frequency band gaps, *New J. Phys.* **20**, 073051 (2018). <https://doi.org/10.1088/1367-2630/aad354>
21. X.K. Han, Z. Zhang, Bandgap design of three-phase phononic crystal by topological optimization, *Wave Motion* **93**, 102496 (2020) <https://doi.org/10.1016/j.wavemoti.2019.102496>
22. Z. Jia, Y. Chen, H. Yang, L. Wang, Designing phononic crystals with wide and robust band gaps, *Phys. Rev. Appl.* **9**, 44021 (2018) <https://doi.org/10.1103/PhysRevApplied.9.044021>
23. X. An, C. Lai, W. He, H. Fan, Three-dimensional meta-truss lattice composite structures with vibration isolation performance, *Extrem. Mech. Lett.* **33**, 100577 (2019) <https://doi.org/10.1016/j.eml.2019.100577>
24. H. Meng, D. Chronopoulos, A.T. Fabro, W. Elmadih, I. Maskery, Rainbow metamaterials for broadband multi-frequency vibration attenuation: numerical analysis and experimental validation, *J. Sound Vib.* **465**, 115005 (2020) <https://doi.org/10.1016/j.jsv.2019.115005>
25. O.R. Bilal, D. Ballagi, C. Daraio, Architected lattices for simultaneous broadband attenuation of airborne sound and mechanical vibrations in all directions, *Phys. Rev. Appl.* **10**, 054060 (2018) <https://doi.org/10.1103/PhysRevApplied.10.054060>
26. Z. Tian, L. Yu, Rainbow trapping of ultrasonic guided waves in chirped phononic crystal plates, *Sci. Rep.* **7**, 40004 (2017) <https://doi.org/10.1038/srep40004>
27. W. Jiang, G. Yin, L. Xie, M. Yin, Multifunctional 3D lattice metamaterials for vibration mitigation and energy absorption, *Int. J. Mech. Sci.* **233**, 107678 (2022) <https://doi.org/10.1016/j.ijmecsci.2022.107678>
28. H. Sheng, M. He, J. Zhao, C. Ting, Q. Ding, P. Lee, The ABH-based lattice structure for load bearing and vibration suppression, *Int. J. Mech. Sci.* **252**, 108378 (2023) <https://doi.org/10.1016/j.ijmecsci.2023.108378>
29. K.K. Saxena, R. Das, E.P. Calius, Three decades of auxetics research – materials with negative poisson’s ratio: a review, *Adv. Eng. Mater.* **18**, 1847–1870 (2016) <https://doi.org/10.1002/adem.201600053>
30. F. Wang, Systematic design of 3D auxetic lattice materials with programmable Poisson’s ratio for finite strains, *J. Mech. Phys. Solids* **114**, 303–318 (2018) <https://doi.org/10.1016/j.jmps.2018.01.013>
31. L. Yang, O. Harrysson, H. West, D. Cormier, Mechanical properties of 3D re-entrant honeycomb auxetic structures realized via additive manufacturing, *Int. J. Solids Struct.* **69**, 475–490 (2015) <https://doi.org/10.1016/j.ijsolstr.2015.05.005>
32. Y. Zhang, L. Sun, X. Ren, X.Y. Zhang, Z. Tao, Y. Min Xie, Design and analysis of an auxetic metamaterial with tuneable stiffness, *Compos. Struct.* **281**, 114997 (2022) <https://doi.org/10.1016/j.compstruct.2021.114997>
33. L. D’Alessandro, V. Zega, R. Ardito, A. Corigliano, 3D auxetic single material periodic structure with ultra-wide tunable bandgap, *Sci. Rep.* **8**, 2262 (2018) <https://doi.org/10.1038/s41598-018-19963-1>
34. A. Seharang, A.H. Azman, S. Abdullah, A review on integration of lightweight gradient lattice structures in additive manufacturing parts, *Adv. Mech. Eng.* **12**, 1–21 (2020) <https://doi.org/10.1177/1687814020916951>
35. X.Y. Zhang, X. Ren, Y. Zhang, Y.M. Xie, A novel auxetic metamaterial with enhanced mechanical properties and tunable auxeticity, *Thin-Walled Struct.* **174**, 109162 (2022) <https://doi.org/10.1016/j.tws.2022.109162>
36. Y. Chen, T. Li, F. Scarpa, L. Wang, Lattice metamaterials with mechanically tunable Poisson’s ratio for vibration control, *Phys. Rev. Appl.* **7**, 024012 (2017) <https://doi.org/10.1103/PhysRevApplied.7.024012>
37. Z. Tao et al., A novel auxetic acoustic metamaterial plate with tunable bandgap, *Int. J. Mech. Sci.* **226**, 107414 (2022) <https://doi.org/10.1016/j.ijmecsci.2022.107414>
38. X. Fei, L. Jin, X. Zhang, X. Li, M. Lu, Three-dimensional anti-chiral auxetic metamaterial with tunable phononic bandgap, *Appl. Phys. Lett.* **116**, 2 (2020) <https://doi.org/10.1063/1.5132589>
39. Y. Wei, Q. Yang, R. Tao, SMP-based chiral auxetic mechanical metamaterial with tunable bandgap function, *Int. J. Mech. Sci.* **195**, 106267 (2021) <https://doi.org/10.1016/j.ijmecsci.2021.106267>
40. M. Kheybari, C. Daraio, O.R. Bilal, Tunable auxetic metamaterials for simultaneous attenuation of airborne sound and elastic vibrations in all directions, *Appl. Phys. Lett.* **121**, 081702 (2022) <https://doi.org/10.1063/5.0104266>
41. D. Qi, H. Yu, W. Hu, C. He, W. Wu, Y. Ma, Bandgap and wave attenuation mechanisms of innovative reentrant and anti-chiral hybrid auxetic metastructure, *Extrem. Mech. Lett.* **28**, 58–68 (2019) <https://doi.org/10.1016/j.eml.2019.02.005>
42. N.H. Vo, T.M. Pham, H. Hao, K. Bi, W. Chen, A reinvestigation of the spring-mass model for metamaterial bandgap prediction, *Int. J. Mech. Sci.* **221**, 107219 (2022) <https://doi.org/10.1016/j.ijmecsci.2022.107219>
43. D. DePauw, H. Al Ba’ba’a, M. Nouh, Metadamping and energy dissipation enhancement via hybrid phononic resonators, *Extrem. Mech. Lett.* **18**, 36–44 (2018) <https://doi.org/10.1016/j.eml.2017.11.002>
44. Y. Huang, J. Li, W. Chen, R. Bao, Tunable bandgaps in soft phononic plates with spring-mass-like resonators, *Int. J. Mech. Sci.* **151**, 300–313 (2019) <https://doi.org/10.1016/j.ijmecsci.2018.11.029>
45. J. Zhao, G. Zhou, D. Zhang, I. Kovacic, R. Zhu, H. Hu, Integrated design of a lightweight metastructure for broadband vibration isolation, *Int. J. Mech. Sci.* **244**, 108069 (2023) <https://doi.org/10.1016/j.ijmecsci.2022.108069>

46. W. Elmadih, D. Chronopoulos, J. Zhu, Metamaterials for simultaneous acoustic and elastic bandgaps, *Sci. Rep.* **11**, 14635 (2021) <https://doi.org/10.1038/s41598-021-94053-3>
47. L. D'Alessandro, R. Ardito, F. Braghin, A. Corigliano, Low frequency 3D ultra-wide vibration attenuation via elastic metamaterial, *Sci. Rep.* **9**, 8039 (2019) <https://doi.org/10.1038/s41598-019-44507-6>
48. M.I. Hussein, R. Khajetourian, Nonlinear Bloch waves and balance between hardening and softening dispersion, *Proc. R. Soc. A Math. Phys. Eng. Sci.* **474**, 20180173 (2018) <https://doi.org/10.1098/rspa.2018.0173>
49. J. Meaud, K. Che, Tuning elastic wave propagation in multistable architected materials, *Int. J. Solids Struct.* **122**, 69–80 (2017) <https://doi.org/10.1016/j.ijsolstr.2017.05.042>
50. A.T. Fabro, H. Meng, D. Chronopoulos, Uncertainties in the attenuation performance of a multifrequency metastructure from additive manufacturing, *Mech. Syst. Signal Process.* **138**, 106557 (2020) <https://doi.org/10.1016/j.ymssp.2019.106557>
51. M. Brun, F. Cortés, J. García-Barruetabeña, I. Sarria, M.J. Elejabarrieta, A robust technique for polymer damping identification using experimental transmissibility data, *Polymers (Basel)*. **14**, 2535 (2022) <https://doi.org/10.3390/polym14132535>

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