

Tunable capacitor arrays of coalesced resonators for dispersion control

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Abstract. In this work, the coupling and dispersion characteristics of coalesced resonators as a function of their capacitance is investigated, with the goal of developing novel ways of dispersion control. When planar resonators are coalesced and their shared side is capacitively loaded, the total coupling coefficient is positive, allowing for the propagation of forward magnetoinductive waves. By varying the capacitive load on their shared side, the sign and size of the total coupling can be controlled. This is demonstrated in an 11-element array, where the magnetoinductive wave can switch between forward and backward propagation depending on the capacitive load of the shared side. Furthermore, there is a critical value of the ratio between the capacitive loads on shared and non-shared sides, at which the coupling becomes zero, effectively cutting off wave propagation on the structure. It is shown that the structure can be tuned in two ways: maintain a constant operating frequency while tuning the coupling, or tune the operating frequency while keeping the coupling constant. At the same time, an optimisation procedure for setting up numerical simulations to match the experimental data is proposed. The simulations provided significant insight on the electric coupling's behaviour. Experimental, numerical and analytical data verify this behaviour.

Keywords: Metamaterials / coupling mechanisms / dispersion / coalesced resonators

1 Introduction

Split ring resonators (SRRs) have garnered considerable attention in research due to their strong magnetic response [1]. Within a single SRR, a circulating current generates a magnetic field perpendicular to the plane of the resonator. When two SRRs are placed in close proximity, they can interact through their magnetic fields, with a time-varying current in one inducing a current in the other through electromagnetic induction. In an array of SRRs, these induced currents propagate along the line like waves. Since the resonators are magnetically coupled, these waves have been termed magnetoinductive (MI) waves [2,3]. MI waves have found applications in imaging [4,5], sensing [6,7], conductivity detection [8], wireless power transfer [9,10] and waveguides [11,12], among others. If the elements are electrically coupled a similar behavior is observed and, in this case, these waves are referred to as electroinductive (EI) waves [13–15].

A SRR can be represented with an electrical circuit equivalent, having a self-inductance from its conductive part and a capacitance at the gap. While the specific shape of the SRR may vary, its response remains consistent

[16–18]. This circuit-based approximation effectively describes the resonance of the SRR when its size is smaller than the wavelength at resonance [19–21]. Hence, it is possible to analytically model a medium composed of coupled SRRs [2,22]. The coupling between the elements enables control over both backward and forward MI waves. Typically, axial arrays facilitate forward-wave propagation because the elements exhibit a positive coupling coefficient, whereas planar arrays support backward-wave propagation due to a negative coupling coefficient between the elements.

The relative orientation of SRRs can influence the coupling mechanisms, as the electric coupling can become stronger when the gaps of neighbouring SRRs are closer [23]. Similarly, the number of gaps has an impact on the coupling [24]. The coupling is also influenced by the frequency. In the THz regime, magnetic coupling considerably weakens, leading to electric coupling being dominant [25,26]. Simultaneously, the coupling coefficients may become complex, causing changes in the band's shape due to retardation effects [12]. Therefore, it is important that the coupling coefficients can be reliably measured. An analytical method for measuring the complex magnetic and electric coupling coefficients from experimental or numerical data is proposed in [27,28]. The method also introduces

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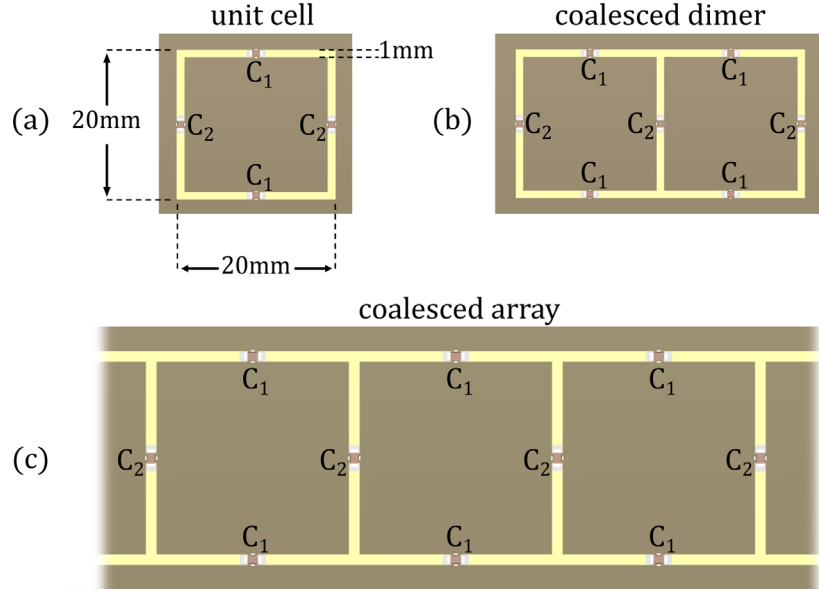


Fig. 1. The four-gap group of PCB samples: (a) The unit cell, (b) Coalesced dimer, and (c) Coalesced 11-element array. Images from KiCad.

two correction factors accounting for the unwanted interactions between the magnetic loop antennas and the resonators, which makes the model more accurate. This method builds on the work of Tatartschuk et al. [26], by fitting a line to the experimental or numerical measurements of two SRRs, whose intercept and slope allow for the extraction of the magnetic and electric coupling coefficients respectively.

While the magnetic and electric couplings between discrete elements has been extensively studied, reports on coalesced resonators have been limited. There are some instances where coalesced SRRs have been explored for applications in optical frequencies [29–32], but the effect on the total coupling is not yet understood. The term conductive coupling was introduced to describe the change in coupling when the elements come into physical contact. In our previous work, it was shown that an analytic model based on circuit theory taking into account mutual inductance and mutual capacitance between elements is fully sufficient to match the experimental and numerical data of coalesced resonators, without the need to introduce the concept of conductive coupling [27,33].

Coalesced resonators lead to interesting results. In a planar array, when the elements are coalesced they can facilitate the propagation of forward MI waves depending on the number of gaps in the SRR [27,33,34]. The key feature here is the side shared among SRRs and whether it is capacitively loaded or not. If the shared side has a gap, the otherwise negligible electric coupling increases in magnitude and the total coupling coefficient between elements switches to positive values. A similar phenomenon emerges when the SRRs are interconnected using capacitive links, thereby forming alternating major and minor loops [35,36]. This configuration essentially generates a diatomic chain of resonators, enabling the tailoring of the dispersion characteristics based on the capacitance

between these two types of loops. Even though diatomic structures of resonators have been shown to be capable of tuning the coupling and dispersion [37–39], coalesced elements are essentially different, as they form a monatomic line with simultaneous electric and magnetic couplings, featuring a single passband.

Here, the coupling mechanisms and dispersion characteristics of coalesced SRRs as a function of their capacitance is investigated. Square resonators with four gaps were considered, where different capacitors were soldered into their gaps. Analytical results are compared with experimental and numerical data. In Section 2, the experimental and numerical setups are summarised. Section 3 presents the results of the unit cell, dimer and array samples for different values of capacitors. Finally, the report ends with the conclusions in Section 4.

2 Experimental and numerical setup

Square resonators with four gaps, as the ones shown in Figure 1, were fabricated on printed circuit boards (PCBs). The group consists of a unit cell (for measuring f_0 , Q , correction factors), a dimer (for calculating κ_H , κ_E) and an 11-element array (for extracting the dispersion). The resonators are square-shaped with side 20 mm, track width 1 mm and gap size 1 mm. The copper tracks have a thickness of 35 μm (1 oz). The PCB dimensions are 100 mm $\times$ 100 mm for the unit cell and dimer, and 270 mm $\times$ 100 mm for the 11-element array, while the thickness of the dielectric substrate in all samples is 1.6 mm. The samples were fabricated on single-layered PCBs with FR4 TG130-140 as the base material. The solder resist layer around the resonators was removed at the design stage, so that capacitors could be soldered at the gaps. On their own, the resonators have a resonance around

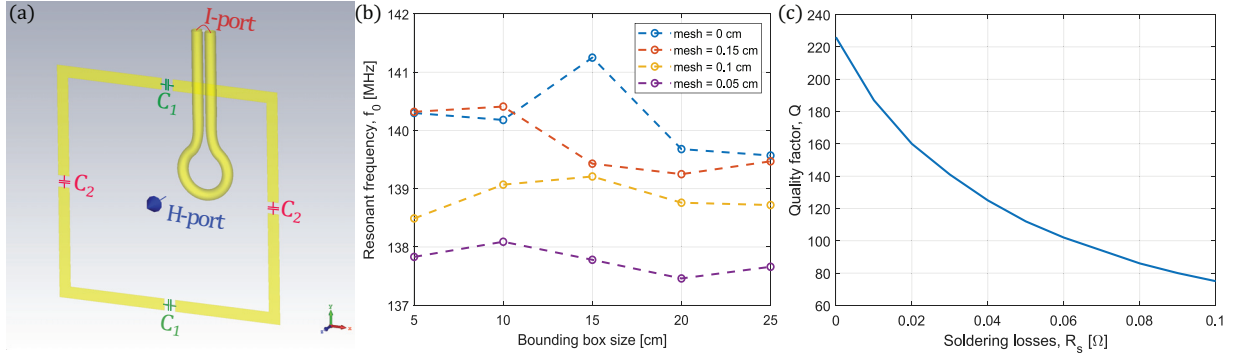


Fig. 2. (a) CST setup for measuring a single SRR. Lumped element capacitors have been placed in all gaps. The SRR is excited with a magnetic loop while the resonance is measured with an H-probe which records the magnetic field; (b) the variation of f_0 at different mesh and boundingbox sizes when $C_1 = C_2 = 100$ pF, and (c) the quality factor Q versus the resistance of each added capacitor.

of 3 GHz. This could be problematic because the wavelength is no longer significantly larger than the unit cell size, an assumption on which the analytical theory is founded on. Hence, for all experiments 100 pF capacitors were soldered at the C_1 gaps. The capacitance C_2 was varied by soldering different capacitors with values of 100 pF, 200 pF and 390 pF. All capacitors were individually measured prior to soldering, so that those within a tolerance of less than 1% of rated values are used.

The measurements were performed using two magnetic loop antennas connected to a vector network analyzer (VNA), as described in [27]. The loops had a diameter of 2.5 mm, wire thickness 1 mm and a gap size 1 mm. The first probe would excite element 1 below its center, while the second probe would move above the center of each element to record the S_{21} parameter. Background measurements without the samples were also recorded and subtracted from the S_{21} during analysis.

The experimental measurement setup can be replicated in Computer Simulation Technology Microwave Studio (CST), see Figure 2a. The dimensions and materials are the same as in the PCB samples and the frequency domain solver was used for the simulations. A copy of the excitation loop used in the experiment was designed in CST with a current port (I-port) placed between its ends. The magnetic field was recorded above the center of each resonator using magnetic field probes (blue arrow). The FR-4 substrate has been made transparent so both the resonator and loop are visible. Lumped element capacitors with the same values as the experimental ones are placed in the gaps. The boundary conditions in all directions are set to open, as they would be in the experimental measurement.

The bounding box size is a parameter that needs to be optimized, since having an infinitely big bounding box is not very efficient. During the simulation, a mesh of the structure inside the bounding box is created by dividing the space into smaller finite elements. CST solves Maxwell's equation for each element and combines them all together afterwards to approximate a solution for the larger structure. Naturally, the size of the mesh will affect the measurement as well. This is clearly shown on Figure 2b, where f_0 changes as the mesh and bounding box sizes are

varied. The resonance frequency measured from the experimental samples can be matched by using an appropriate combination of mesh and bounding box sizes. The quality factor (not shown) had a similar response, but was significantly higher than the experimentally measured Q . Initially, CST computes ohmic, radiative and dielectric losses. However, in the experiment there are additional losses introduced due to the soldered capacitors, R_s . Increasing the resistance of the lumped element capacitors in CST, results in a decrease in Q , see Figure 2c. By matching the quality factor, not only the simulation becomes more accurate but R_s can also be estimated.

3 Results

3.1 Coupling coefficients

The normalised experimental $|S_{21}|$ parameter (solid blue line) and numerical $|H_1|$ field (dashed blue line) for the unit cell with 100 pF capacitors in all gaps are shown in Figure 3a. The agreement between the numerical and experimental data is excellent. In CST, the mesh and bounding box sizes used are 0.128 cm and 10 cm respectively. The self inductance of the unit cell was found to be 51.6 nH, while the radiative, ohmic and dielectric resistances were calculated as $R = 0.21 \Omega$. Finally, the soldering losses needed to match the quality factor are $R_s = 0.248 \Omega$, shared between all capacitors of the unit cell.

Keeping the mesh, bounding box and soldering losses the same, the agreement between experiment and simulation at the dimers appears to slightly falter, as a frequency mismatch emerges, see Figure 3b. Our hypothesis is that CST produces an approximation for each geometry by creating a mesh. Although the simulation was fitted to the experimental data of the unit cell, when a second element is added the geometry is changed and so is the meshing. Regardless, the agreement remains good overall and the simulation parameters are not changed. There is a resonance split when 100 pF and 390 pF capacitors are soldered in the C_2 gaps. However, when 200 pF are soldered in the C_2 gaps, the resonance split is significantly weaker, showing signs of a near-zero total coupling between the elements.

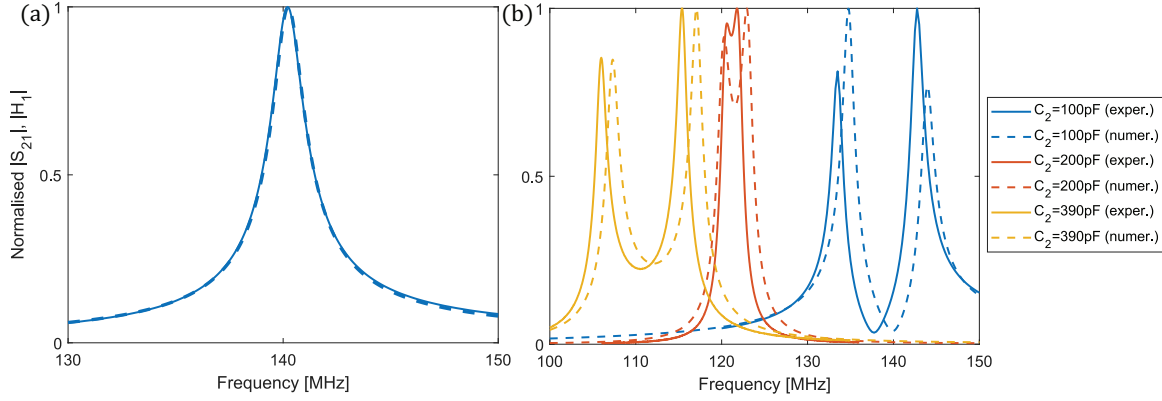


Fig. 3. Normalised experimental $|S_{21}|$ parameter (solid lines) and numerical $|H_1|$ field (dashed lines). (a) For the unit cell with 100 pF capacitors in all gaps, and (b) for coalesced dimer samples with different capacitors in the C_2 gap.

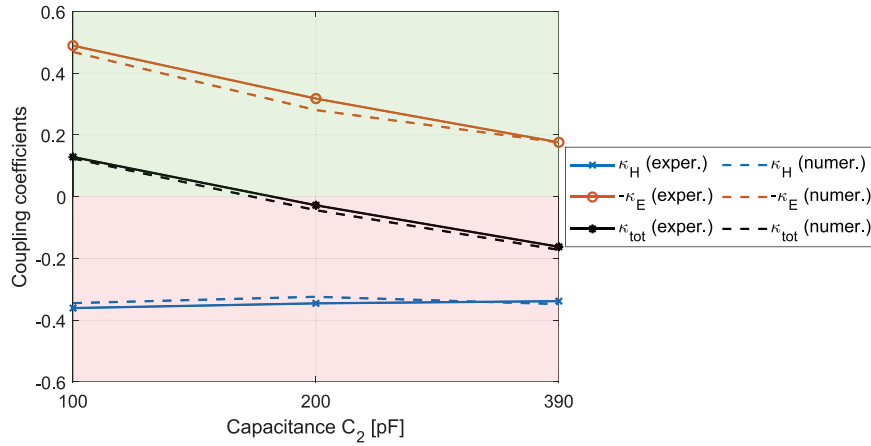


Fig. 4. The coupling coefficients extracted from experimental (solid lines) and numerical (dashed lines) data for 100 pF, 200 pF and 390 pF capacitors in the C_2 gaps.

If L , C are the self-inductance and capacitance of one SRR, and M, K , are the mutual inductance and capacitance between two SRRs, the magnetic and electric coupling coefficients can be defined as

$$\kappa_H = \frac{2M}{L} \text{ and } \kappa_E = \frac{2C}{K}. \quad (1)$$

The coupling coefficients can be extracted using experimental or numerical data from

$$\kappa_H - \frac{f_0^2}{f^2} \kappa_E = -2 \frac{\frac{I_2}{I_1} - \mu}{1 - \mu \frac{I_2}{I_1}} \left(1 - \frac{f_0^2}{f^2} - j \frac{f_0}{fQ} \right), \quad (2)$$

where f_0 and Q are the resonance frequency and quality factors of a single SRR, f is the frequency, $I_{1,2}$ are the currents of each SRR, and μ is a correction factor associated with the measurement setup [27]. There is also a correction factor v , which is not shown in equation (2), but is incorporated into the currents $I_{1,2}$. The two coupling coefficients can reinforce or counteract each other depending on their sign. The behaviour of the structure can be predicted by using the total coupling

Table 1. Mutual capacitance.

C_2 [pF]	C [pF]	κ_E	K [pF]
100	25	-0.47	-106
200	33	-0.30	-222
390	40	-0.18	-444

coefficient which is defined as

$$\kappa_{tot} = \kappa_H - \frac{f_0^2}{f^2} \kappa_E \sim \kappa_H - \kappa_E. \quad (3)$$

Using $v = -0.07$ and $\mu = -0.1$, κ_H and κ_E can be calculated separately from equation (2), as shown in Figure 4. When $C_2 = 100$ pF, κ_H is negative and $-\kappa_E$ is positive, so that the total coupling coefficient is positive. As the capacitance in C_2 increases, κ_E weakens while κ_H remains roughly the same. This results in κ_{tot} to switch to negative values, passing through zero at around $C_2 = 200$ pF, which was expected from the dimer fields in Figure 3b.

Using the definition for the electric coupling coefficient, see equation (1), the mutual capacitance K can be calculated at different values of C_2 . From Table 1, it can

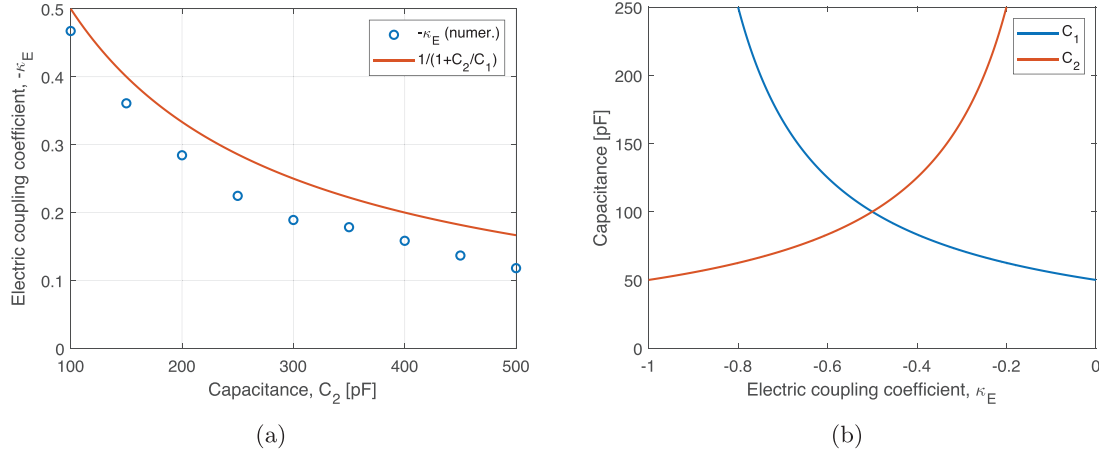


Fig. 5. (a) Comparison of κ_E extracted from numerical data and equation (4) for different values of C_2 , and (b) C_1 , C_2 from equations (6) and (7) that produce different values of κ_E while maintaining a constant total capacitance.

be seen that $K \sim -C_2$ across all cases measured and the electric coupling coefficient becomes

$$\kappa_E = -\frac{2}{C_2} \frac{C_1 C_2}{2C_1 + 2C_2} = -\frac{C_1}{C_1 + C_2} \Rightarrow \kappa_E = -\frac{1}{1 + \frac{C_2}{C_1}}. \quad (4)$$

Using CST the coupling coefficients for more values of C_2 can be calculated, shown as blue circles in Figure 5a. The red line calculated from equation (4) is slightly higher than the extracted values of $-\kappa_E$, but is in a good agreement overall.

Equation (4) can be substituted into equation (3) to acquire an expression for the capacitance ratio,

$$\begin{aligned} \kappa_{tot} &= \kappa_H + \frac{1}{1 + \frac{C_2}{C_1}} \Rightarrow 1 + \frac{C_2}{C_1} = \frac{1}{\kappa_{tot} - \kappa_H} \\ &\Rightarrow \frac{C_2}{C_1} = -\left(1 + \frac{1}{\kappa_H - \kappa_{tot}}\right). \end{aligned}$$

Assuming that $\kappa_H = -0.35$, the zero coupling condition ($\kappa_{tot} = 0$) is shown to be

$$\frac{C_2}{C_1} = 1.86. \quad (5)$$

In our case of $C_1 = 100$ pF, when $C_2 = 186$ pF the total coupling should be zero, which appear to be the case from Figure 4. If the capacitance ratio is below 1.86, the total coupling coefficient is positive, allowing for forward waves to propagate on the structure. Conversely, when the capacitance ratio is over 1.86, the total coupling coefficient is negative and backward waves can propagate. Furthermore, equation (5) implies that C_1 , C_2 can be changed in such a way to tune f_0 , while their ratio stays the same and thus have the same total coupling between elements. The inverse scenario is even more interesting: C_1 , C_2 can be changed to tune κ_{tot} while f_0 does not change. From equation (1) with $K \sim -C_2$

$$\kappa_E = -\frac{2C}{C_2} \Rightarrow C_2 = -\frac{2C}{\kappa_E}. \quad (6)$$

Now the total capacitance can be rewritten as

$$\begin{aligned} C &= -\frac{C_1 \frac{2C}{\kappa_E}}{2C_1 - 2\frac{2C}{\kappa_E}} \Rightarrow C_1 - \frac{2C}{\kappa_E} = -\frac{C_1}{\kappa_E} \\ &\Rightarrow \kappa_E C_1 - 2C = -C_1, \end{aligned}$$

which can be solved for C_1

$$C_1 = \frac{2C}{1 + \kappa_E}. \quad (7)$$

Assuming that the self-inductance of the SRR is unaffected by changing the capacitances C_1 and C_2 , for the resonance to be the same, C needs to be a constant. Then in equations (6) and (7) the only variable is κ_E , so there is a combination of C_1 and C_2 that can produce a range of electric coupling coefficients while the resonance does not change. In Figure 5b, C_1 and C_2 for different values of κ_E with $C = 25$ pF are shown.

3.2 Dispersion

Equipped with the knowledge of all system parameters ($f_0, Q, \kappa_H, \kappa_E$), the 11-element array can now be investigated. The goal is to evaluate the dispersion characteristics of the propagating MI wave and verify whether it can be switched between forward and backward by adjusting the capacitance of the shared sides. Assuming wave propagation in the periodic structure, the current of element m is

$$I_m = I_0 e^{-jmkd},$$

where I_0 is the wave's amplitude, $k = \beta - ja$ is the wave number with β and α being the dispersion and attenuation coefficients respectively, and d is the unit cell size. The currents of any three consecutive elements I_{m-1} , I_m , I_{m+1} are connected through trigonometry as

$$I_{m-1} + I_{m+1} = 2I_m \cos kd. \quad (8)$$

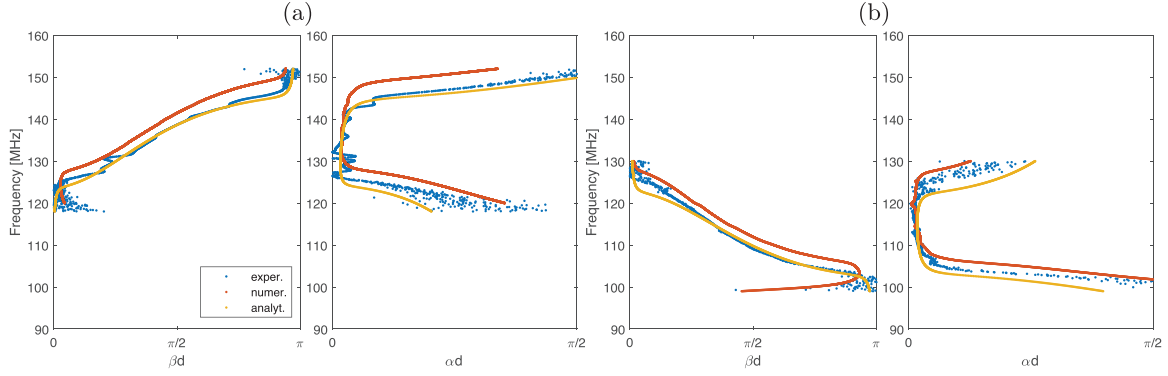


Fig. 6. Dispersion with (a) $C_2 = 100$ pF, and (b) $C_2 = 390$ pF. The experimental (blue dots) and simulation (red dots) data were calculated from the currents of three consecutive elements. The analytic model (yellow line) includes losses and retardation.

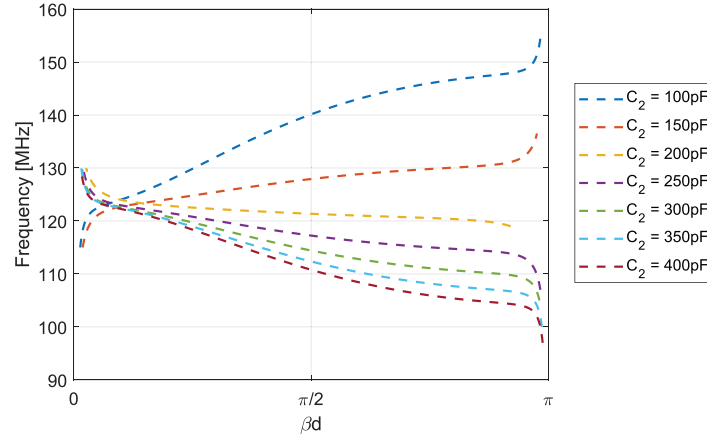


Fig. 7. Evolution of the dispersion as C_2 increases.

It can be shown that the dispersion can be calculated analytically from [27]

$$\cos kd = \cos(\beta d - j\alpha d) = -\frac{1 - \frac{f_0^2}{f^2} - j\frac{f_0}{Qf}}{\left(\kappa_H - \frac{f_0^2}{f^2}\kappa_E\right)}. \quad (9)$$

The dispersion can be plotted analytically through equation (9) by knowing the resonance, quality factor and coupling coefficients, and through equation (8) by using the experimental or numerical measurements of three consecutive resonators.

In Figure 6 the dispersion for $C_2 = 100$ pF and 390 pF are plotted. Blue dots are from experimental data, red dots are from numerical data and the yellow line is from the analytical model including losses and retardation. The agreement between experimental data and analytic model is excellent. There is a frequency mismatch with the numerical data, which had already arisen from the dimer samples. The experimental and numerical extraction relies on the magnetic field of each resonator. In all

cases, the frequencies outside the passband have relatively low magnetic fields, making the extracted dispersion unreliable in these parts, which have been removed. For $C_2 = 200$ pF, the passband is very narrow and not clearly visible when using experimental or numerical data to plot it, hence the dispersion for this case is not shown.

As shown on Figure 6a, when $C_2 = 100$ pF there is forward wave propagation with a passband of roughly 30 MHz. However, when $C_2 = 390$ pF there is backwards propagation with a similar band size. Not only the propagating wave can be switched between forward and backward, but the size of the bandgap can be tuned by controlling C_2 . In Figure 7 the evolution of the analytic dispersion as the capacitance C_2 increases is shown. In all cases $C_1 = 100$ pF, so, from equation (5), when C_2 is less than 186 pF there is a forward wave propagating on the structure. The size of the passband shrinks as C_2 increases, until it switches to backward wave propagation for values over 186 pF. At $C_2 = 200$ pF, the passband is almost non-existent, because the ratio of C_2/C_1 is around the critical value of zero coupling where no propagation occurs.

4 Conclusions

A planar array of coalesced square resonators with tunable dispersion characteristics was demonstrated. The study revealed that the magnitude and sign of the total coupling coefficient between elements can be tuned by controlling the capacitive loads of the shared gaps C_2 . It was concluded that the magnetic coupling remains unaffected by variations in C_2 , but, as C_2 increases for a constant value of C_1 , the electric coupling weakens. Hence, in an array of resonators the propagating magnetoinductive wave can be switched between forward and backward. This behaviour was verified with an analytic model, experimental data and numerical simulations.

The optimised methodology of fitting numerical simulations to experimental data enabled the investigation of the electric coupling's behaviour. This approach aims to reduce computational resource requirements while ensuring the reliability of the simulations. It was shown that the magnitude of the mutual capacitance between resonators K is approximately equal to the capacitive load on the shared side. This allowed the derivation of analytic expressions for the electric coupling coefficient κ_E and the total coupling coefficient κ_{tot} , which showed good agreement with the corresponding extracted values. Furthermore, it was found that there is a critical value of the capacitance ratio, C_2/C_1 , at which the electric coupling cancels out the magnetic coupling, making the total coupling between elements zero, switching off wave propagation entirely. Below this critical value, the total coupling coefficient is positive and an array will support forward MI waves, while above this critical value the total coupling coefficient is negative and backward MI waves will propagate. It was demonstrated that the structure can be tuned to either maintain a constant operating frequency with a tunable total coupling or have a tunable operating frequency at a constant total coupling coefficient.

These results would be very interesting to expand in two dimensions and three dimensions, by coalescing along on or more axes, thus offering a new degree of flexibility to the designer. For example, the total coupling coefficient could be tuned separately in the x - and y - directions, allowing for advanced ways of dispersion control. Even though this behaviour was demonstrated with capacitive loads of a few hundred pF, the same effect could be replicated with different values of C_1 and C_2 .

In practise, the capacitance on the shared side could be controlled with liquid crystals, which are capable of changing the capacitance in response to voltage or temperature variations. This poses the challenge of creating a design to incorporate low-loss liquid crystals within the structure. Such devices could be used in applications where wave control is important. At the same time, they could be used as environment sensors, by retrieving the temperature from the bandpass type and size, or switches, where wave propagation is switched off when the temperature reaches a certain value. In the future, it would be interesting to investigate capacitance variations in specific gaps of the resonators, to achieve similar or different effects of dispersion control.

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Conflicts of interest

The authors have no conflicts to disclose.

Data availability statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Author contribution statement

I.S., C.J.S. and E.S. contributed to the project conception. I.S. and C.J.S. performed the experiments. I. S. did the simulations, data analysis and wrote the initial version of the manuscript. All authors contributed with discussions on the analytic model, the results and writing the final version of the manuscript.

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