

Design method for large-scale wide field-of-view monochromatic metalenses

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Abstract. We propose a novel design method for wide field-of-view monochromatic metalenses. The proposed technique partitions the outer region of a metalens into supercells, which are generated by dividing the outer region into intervals along the radial direction, where the target phase changes by 2π , and along the angular direction with a constant angular periodicity. Therefore, the shape of each supercell can be approximated as a rectangle with its size comparable to a wavelength. The arrangement of pillars within this supercell is determined by metagrating optimization via the adjoint method. The optimization process considers both inter-pillar couplings and the range of incidence angles. This makes the design of large-scale wide field-of-view high-efficiency metalenses more tractable than the conventional unit-cell-based method, which is prone to efficiency decrease especially near the lens periphery. Furthermore, it has a potential advantage in terms of computational cost over other recently proposed optimization-based methods.

Keywords: Metasurface / metalens / metagrating

1 Introduction

Monochromatic metalenses are garnering significant attention as the demand for compact sensing devices increases. They are typically designed using the unit-cell-based method [1–4], which is computationally efficient. However, this method relies on the meta-atom library, which is the database of characteristics of meta-atoms in a periodic array. Its efficiency decreases due to deviation from local periodic approximation (LPA), which is particularly remarkable near the edge of the large-scale wide field-of-view metalens where the phase changes rapidly in space.

Recently, some studies [5–8] have reported the application of optimization-based methods to design various metasurfaces. These methods have the potential for novel, high-performance designs, leveraging their flexible degrees of freedom and handling of inter-pillar coupling. They consider the entire metasurface or its subregions, which are significantly larger than the wavelength, as optimization regions. At each iteration, they run full-wave simulations across large optimization regions to employ a gradient-based algorithm utilizing the adjoint technique [9–12] or a genetic algorithm.

Consequently, in the optimization demonstrations in these studies, the size has been limited to $\sim 100\lambda$ for 2D metalenses (cylindrical metalenses) [5,6,8] and $\sim 50\lambda$ for 3D metalenses [6,7]. It is currently challenging to apply these optimization-based methods to practical 3D metalenses larger than 1000λ , such as those discussed in the Results section, due to the high computational cost.

Other studies [13–15] have proposed an alternative approach for metalens design. This approach involves dividing the metalens into smaller computational areas, known as supercells, and designing metagratings within each supercell before assembling them to form the complete lens. This approach leverages the idea that the phase profile, moving away from the center of the lens, can be effectively depicted as an assembly of metagratings. The optimization techniques used for these metagratings have advanced by incorporating modern technologies such as genetic algorithms [16] and machine learning [17]. This approach enables the design of large metalenses within feasible computational constraints. However, these studies presuppose scenarios where each position on the metalens corresponds to a single incident angle. This assumption is not adequate for wide field-of-view monochromatic metalenses, where even a single position on the metalens is characterized by a range of incident angles. Consequently, to effectively adapt this approach for wide field-of-view

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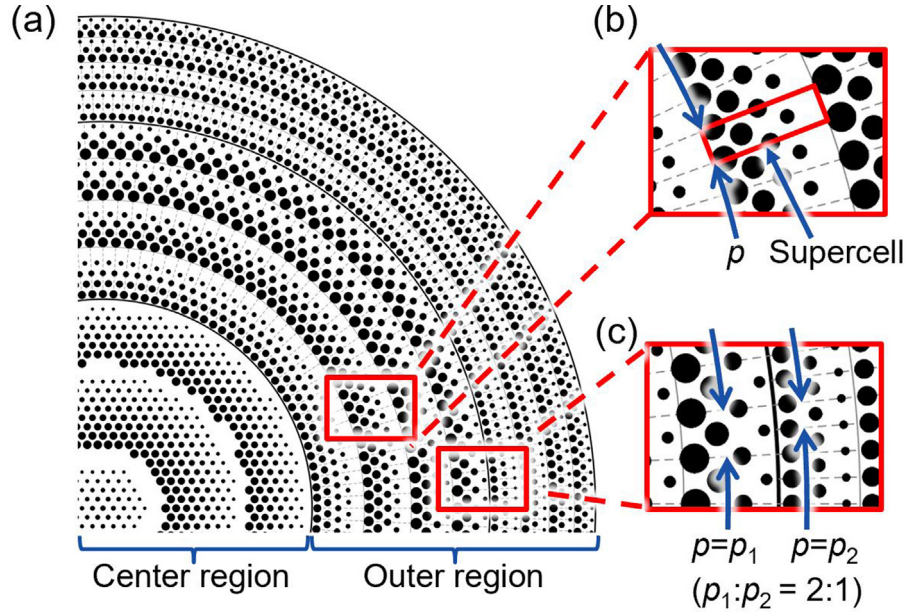


Fig. 1. Schematic of the metalens designed by the proposed method. (a) Entire metalens is divided into the center and outer regions. (b) Supercell with intervals along the radial direction, where the target phase changes by 2π , and with intervals along the circumferential direction which have a constant angular period p . (c) Particular point where the angular period is adjusted to a smaller value.

monochromatic metalenses, the optimization framework must be revised. It should incorporate considerations for the range of incident angles at each position on the metalens.

2 Method

2.1 Overall strategy

The schematic of a metalens designed by our method is illustrated in Figure 1. Similar to a previous study [13], the metalens is divided into the center and outer regions (Fig. 1a). The conventional unit-cell-based design method is applied to the center region because it does not cause a significant reduction in efficiency near the lens center. The outer region is further partitioned into supercells. These are generated by dividing the outer region into intervals along the radial direction, where the target phase changes by 2π , and along the angular direction, which have a constant angular period p . It is preferable to maintain a constant angular period along the radial direction to ensure the LPA of the supercell (Fig. 1b). However, it is acceptable to adjust the angular period to a smaller value at particular points to maintain the circumferential width of the supercell below the wavelength and suppress unwanted diffraction from the pillar pattern (the features described so far are similar to those in a previous study [13]). Specifically, we reset the angular period with a ratio defined by two small integers. An example of this is shown in Figure 1c, where a 2:1 ratio is employed. Generally, the pillar pattern across this reset boundary exhibits an angular period equal to the least common multiple of the two angular periods inside and outside the boundary. Therefore, employing a 2:1 reset ratio, we can realize the

highest angular periodicity, leading to reduced complexity in the pillar pattern across the boundary. This approach could be advantageous for semiconductor fabrication by improving the manageability of the optical proximity correction process. This correction process is pivotal for maintaining design fidelity in the photolithographic patterning. This approach also assists in verifying the phase continuity at the boundary, marking a significant distinction from other supercell-based design methods for large 3D metalenses in references [13,14], which lack a straightforward method to confirm such phase continuity. By properly setting the angular period and boundary between the center and outer regions, the shape of the supercell is effectively approximated as a rectangle. Each supercell acts as a metagrating and has a pillar arrangement that can be designed by solving a metagrating optimization problem. Similar to most other optimization-based methods, our approach uses the adjoint technique for optimization to account for inter-pillar coupling, which contributes to maintaining high efficiency in the outer region of the lens. Distinctively, our proposed method uses rigorous coupled-wave analysis (RCWA) [18] for forward/adjoint simulations, whereas most other optimization-based methods employ the finite-difference time-domain (FDTD) method [19]. Additionally, the radial width of the supercell is comparable to the wavelength. Consequently, our method has a moderate computational cost. As described in the next subsection, the optimization of each supercell considers the range of incident angles. This is the second major difference from the design methods in references [13–17], which assume a single incident angle at each position on the metalens. The applications of these methods are typically limited to metalenses that either

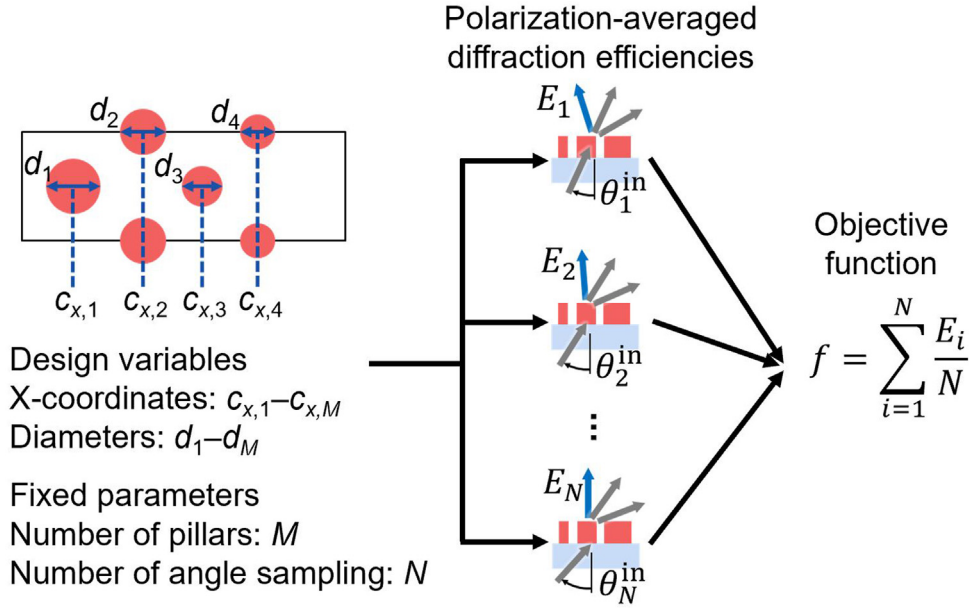


Fig. 2. Problem setup of metagrating optimization for supercell pillar pattern.

focus normally incident light into a focal point or collimate light from a point source. In contrast, our method targets wide field-of-view monochromatic metalenses, where each supercell has a range of incident angles. Therefore, in designing such lenses, it is essential to identify a metagrating design for each supercell that operates robustly within its specified range of incident angles.

2.2 Framework for metagrating optimization

The problem setup of the metagrating optimization for the supercell pillar pattern is illustrated in Figure 2. We aim to achieve robustness against angle and polarization by properly defining the optimization problem. We seek the optimal pattern among the zigzag-arrangement of circular pillars. Thus, the design variables are pillars' local x -coordinates $c_{x,1}-c_{x,M}$ and diameters d_1-d_M . Here, M is the number of pillars. The objective function to be maximized is the average minus first-order diffraction efficiency for the angle and polarization obtained by RCWA. Incidence angles are sampled from the possible range for the supercells. So far, we have only accounted for meridional rays that have zero-azimuth and non-zero-zenith incident angles. To ensure the manufacturability, we impose several fabrication constraints on the pillar arrangement. Figure 3 illustrates several dimensions to be constrained in the case of pillar number $M = 4$. The spaces between adjacent pillars s_1-s_{3M} must be bigger than the predefined minimum space $s_{\min}$. The distances between centers of adjacent pillars l_1-l_{2M} must be larger than the predefined minimum distance $l_{\min}$. The diameters of pillars d_1-d_M must be bigger than the predefined minimum diameter $d_{\min}$ and smaller than the predefined maximum diameter $d_{\max}$. We define constraint functions g_1-g_{7M} . The negative constraint function indicates that the corresponding constraint is satisfied.

To address this optimization problem with inequality constraints, we apply a gradient-based algorithm using the adjoint technique. At each iteration, the derivative of the objective function with respect to refractive index distribution is evaluated by the forward/adjoint RCWA simulations [20–22]. To transform this derivative into the derivative of the objective function with respect to the design variables using the chain rule, we introduce the following differentiable formulation, which approximates the refractive index distribution:

$$n(x, y; \beta) = \sum_{k=1}^M T(D(x, y; c_{x,k}, c_{y,k}, d_k); \beta),$$

where

$$D(x, y; c_x, c_y, d) = \frac{d}{2} - \sqrt{(x - c_x)^2 + (y - c_y)^2},$$

$$T(\alpha; \beta) = \frac{n_{\text{pillar}} - n_{\text{encapsulant}}}{2} \tanh(\beta\alpha) + \frac{n_{\text{pillar}} + n_{\text{encapsulant}}}{2}.$$

Here, $c_{y,k}$ denotes the local y -coordinate of the k -th pillar; $D(x, y; c_x, c_y, d)$ is the signed distance between a point (x, y) and a circle defined by its center (c_x, c_y) and diameter d ; $T(\alpha; \beta)$ is the threshold function, mapping the input α into the output ranging from the refractive index of encapsulant $n_{\text{encapsulant}}$ to refractive index of pillar n_{pillar} ; β serves as an iteration-dependent parameter regarding the sharpness of the threshold function. As depicted in Figure 4, with the ascent of β , $n(x, y; \beta)$ converges toward the binarized refractive index distribution. This approach of derivative computation via a quasi-binary refractive index distribution is inspired by the sensitivity analysis technique employed in topology optimizations for photonic

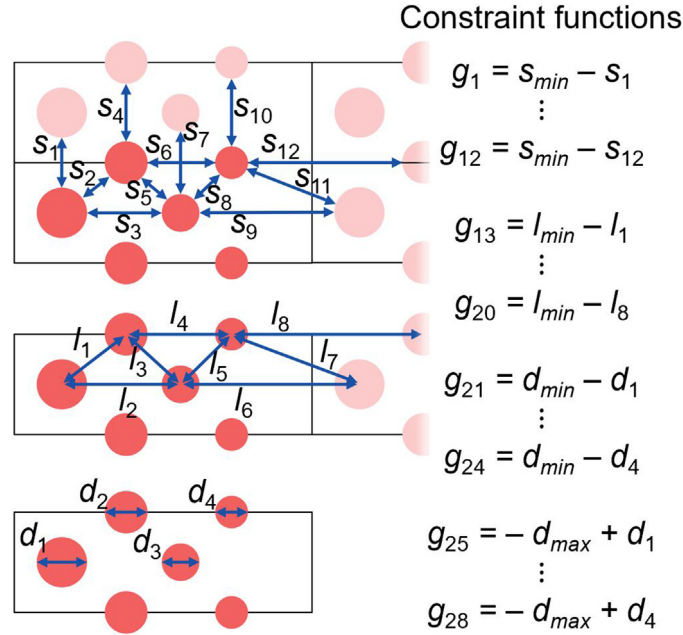


Fig. 3. Dimensions to be constrained during optimization in the case where the number of supercell pillars is $M = 4$.

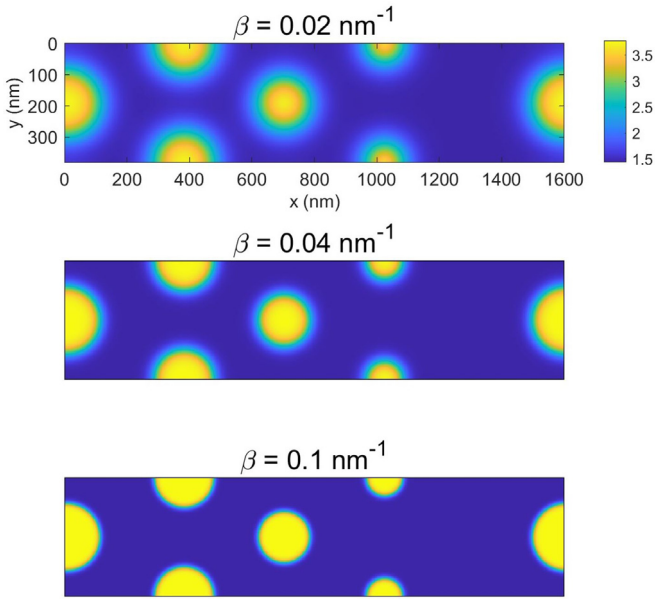


Fig. 4. Quasi-binary refractive index distribution $n(x,y;\beta)$ and its dependence on the sharpness parameter β . As β increases, $n(x,y;\beta)$ approaches the binarized refractive index distribution.

crystal waveguides and metagratings, which involves the utilization of a comparable threshold function [10,20–22]. Hereafter, the phrase “with respect to the design variable” is omitted for brevity. If the update of design variables along the objective function’s derivative satisfies all constraint functions, this update direction is adopted. Conversely, if this update violates certain constraint functions, the update direction is determined using the derivatives of

the constraint functions. If $\mathbf{a}$ represent the derivative of the objective function, and $\mathbf{q}_1\text{--}\mathbf{q}_m$ represent the derivatives of these particular constraint functions. Note that $\mathbf{q}_1\text{--}\mathbf{q}_m$ do not necessarily correspond to the derivatives of the constraint functions $g_1\text{--}g_m$, respectively. Instead, they represent the derivatives of those constraint functions that are violated following the update of design variables along the derivative of the objective function. These particular constraint functions are just a subset of all the constraint functions. For example, in the optimization results shown in Figure 5b, beyond the 11th step, the number of such relevant constraint functions is only $m = 1$, where $\mathbf{q}_1$ corresponds to the derivative of g_{24} . The update direction $\mathbf{b}$ is calculated by projecting $\mathbf{a}$ onto a subspace orthogonal to the subspace spanned by $\mathbf{q}_1\text{--}\mathbf{q}_m$, using the following linear algebra formula:

$$\mathbf{b} = \{I - A(A^T A)^{-1} A^T\} \mathbf{a},$$

where $A = [\mathbf{q}_1 \mathbf{q}_2 \dots \mathbf{q}_m]$. This update direction increases the objective function while keeping all the constraints satisfied. By iteratively updating the design variables using the update direction determined in this way, we can obtain a high-performance design that is robust against polarization and incidence angle (Fig. 5a), while satisfying all the constraints (Fig. 5b).

2.3 Sequential and selective optimization approach for determining comprehensive pillar patterns

By applying the metagrating optimization to the supercells sequentially from the inside (or outside) along the radial direction, the entire pillar pattern in the outer region can be determined. During this procedure, the optimization results from one supercell can be inherited as the initial

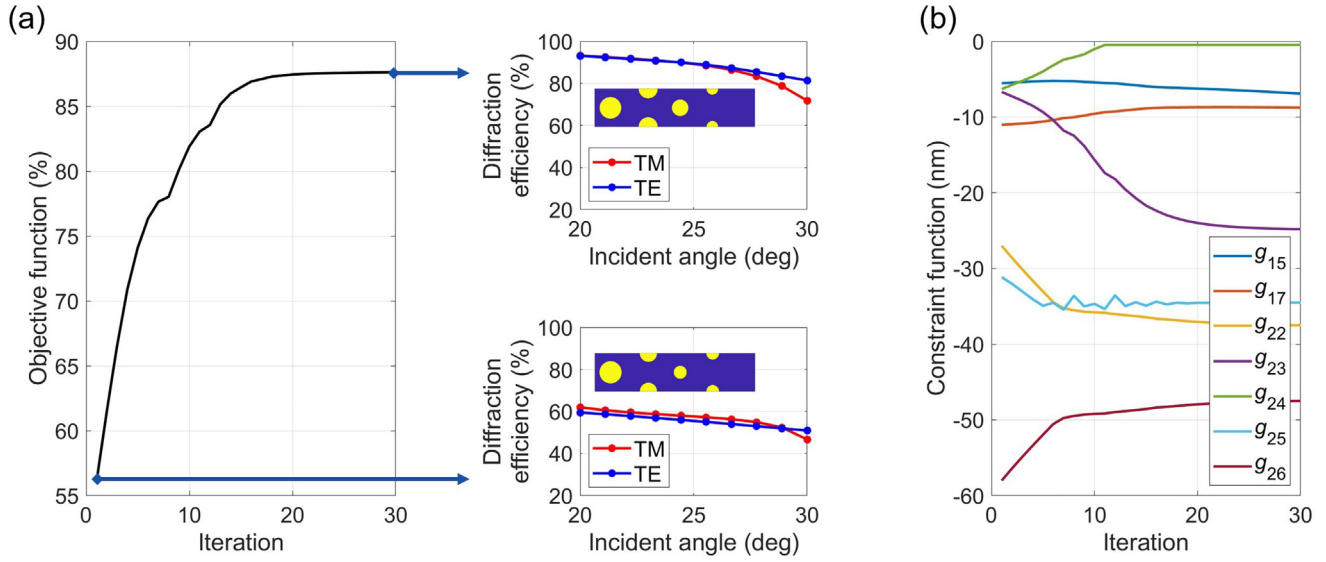


Fig. 5. (a) Objective function over the course of optimization. (b) Constraint functions over the course of optimization. Only the top seven constraint functions ranked in descending order by their values at the final step are plotted.

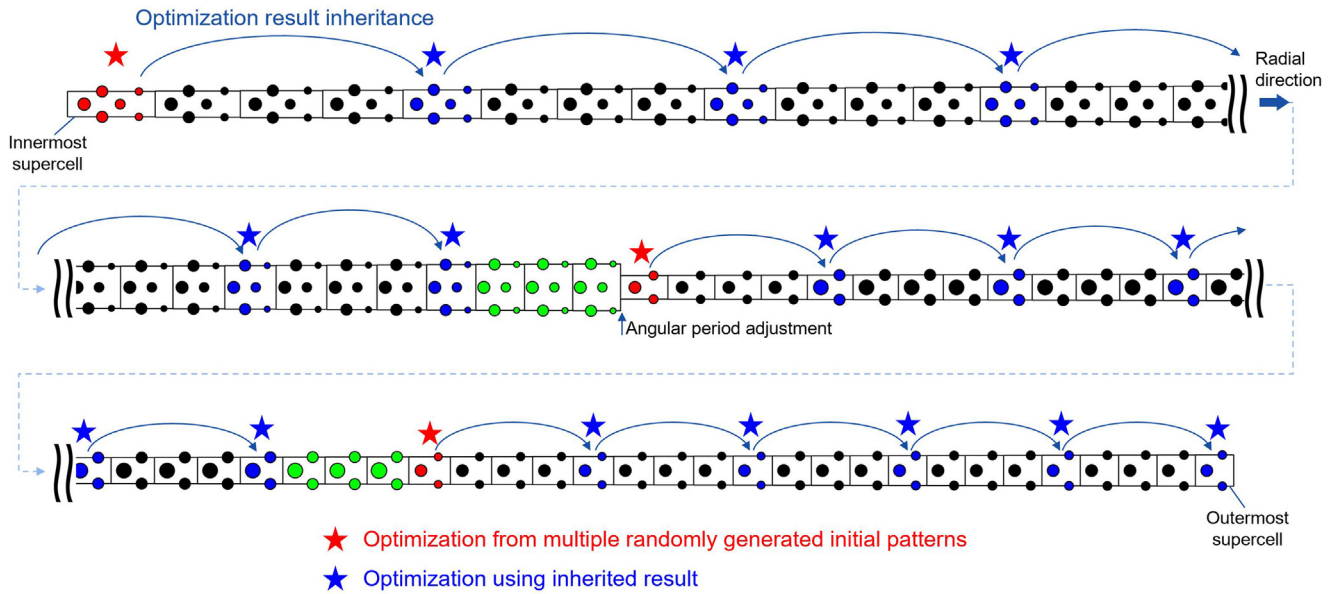


Fig. 6. Schematic of sequential and selective optimization procedure for determining comprehensive pillar patterns.

values for the next optimization, allowing the computation to be executed efficiently. This also leads to pattern similarities between neighboring supercells, thus ensuring the validity of applying metagrating optimization to each supercell. This application implicitly assumes the LPA of each supercell.

The procedure to determine the entire pillar patterns is illustrated in Figure 6. First, for the innermost supercell, the metagrating is optimized from multiple random initial patterns. The optimization result with the highest objective function is chosen as the pillar arrangement in the innermost supercell and is inherited as the initial design

for the subsequent optimization. While it is possible to perform optimization on all other supercells subsequently, it is computationally cost-effective to apply optimization only to selected supercells at specific intervals. Figure 6 illustrates the case where the supercells at intervals of four (labeled by blue star marks) are optimized using inherited results. Notably, in supercells where the angular period is adjusted or where inheriting the optimization results no longer enables the maintenance of a high objective function (labeled by red star marks except for the innermost one), the optimization from random initial patterns is performed again. After finishing optimizations for all the supercells

selected at specific intervals, the pillar patterns in the remaining supercells are determined by either interpolation or extrapolation. The interpolation from the nearest inner and outer optimized supercell is executed. Notably, in the case of supercells denoted by green pillars, when the nearest outer supercell has a pillar arrangement obtained through optimization from random initial patterns, the pillar patterns are determined by extrapolation from the nearest inner supercell.

3 Results

3.1 Conventional design

For comparison with our proposed design, we initially employed the conventional unit-cell-based method to design a metalens at a wavelength of 940 nm with a field-of-view of 131° , f -number of 1.6, and diameter of over 2 mm. This lens features a metasurface comprising the amorphous-silicon circular pillars with a height of 600 nm on the glass substrate. Its layout is illustrated in Figure 7. Figure 8 illustrates the target phase optimized by the commercial optical design software (CODE V) and the range of incident angles in the glass substrate. As depicted in Figure 7, rays entering from the air at a maximum angle of $\varphi_{\max} = 65.5^\circ$ and reaching the maximum image height strike the metasurface within the substrate at an angle of $\theta_{\max} = 38.8^\circ$. At any given point (except for the outermost point) on the metasurface, rays reaching different image heights pass through. Therefore, as illustrated in Figure 8, the incident angle at that position is not singular but spans a finite range. The upper limit of this incident angle range becomes $\theta_{\max}$ at positions on the metasurface that are outward from the point marked with a star in Figure 8, which is the innermost position among those traversed by rays reaching the maximum image height as depicted in Figure 7. Pillars of diameter from 100 to 280 nm are arranged hexagonally, and the diameter of the hexagonal unit-cell is 380 nm.

This paragraph outlines the methodology employed to evaluate the focusing performance of metalens for meridional rays using a simplified simulation. Given our interest in wide field-of-view metalenses, we consider not only the normal incident light as simulated in reference [13] but also various oblique incident lights that focus at different image heights. Whereas reference [4], which employs the unit-cell-based method, evaluates the focusing efficiency for a wide field-of-view metalenses at multiple incident angles using FDTD, our case presents more complexity. The area covered by the pillars and the aperture of our metalens is over ten times larger than those in the reference, making the use of FDTD impractical. Therefore, we assessed the focusing efficiency for meridional rays using a combination of RCWA and ray tracing. Due to the mismatch between the uniform unit-cell period of the conventional method and the periodicity of the target phase, we cannot freely define regions where RCWA can be applied. Nonetheless, it is possible to discretely identify specific regions that exhibit local periodicity. In Figure 9, we present a plot of the normalized grating pitch, denoted as G . This represents the length of the interval

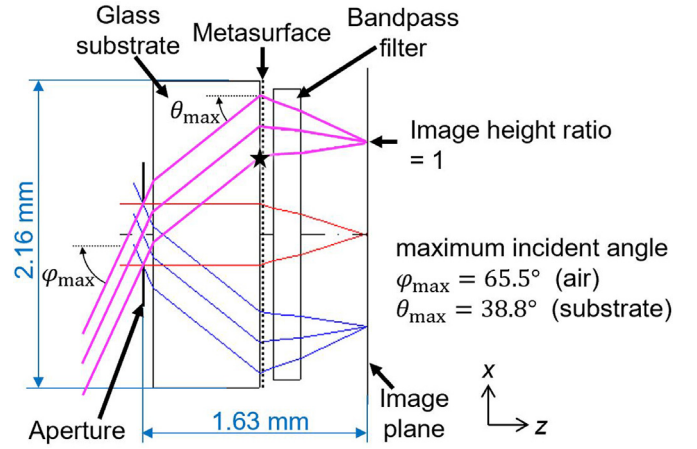


Fig. 7. Layout of metalens designed as a proof of concept.

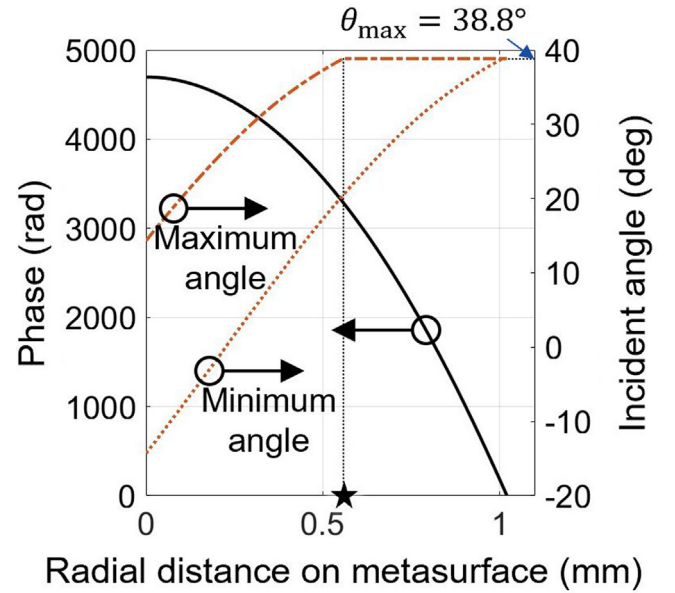


Fig. 8. Target phase optimized by commercial optical design software and incident angle range.

along the x -axis over which the target phase undergoes a 2π change, normalized with respect to the x -directional period of the unit-cell (380 nm). We identify several positions where G can be expressed as a ratio a/b (where a and b are integers). In the vicinity of such positions, local periodicity exists, as the phase changes by $2\pi \times b$ over an interval of a unit-cells aligned along the x -axis. An example of this is the pillar arrangement near $x = 811 \mu\text{m}$, where G equals $9/4$, as shown in Figure 10. The arrangement of pillars within the section including nine unit-cells (red frame) exhibits high similarity to that within the adjacent (blue frame). We conducted RCWA simulations for the locally periodic

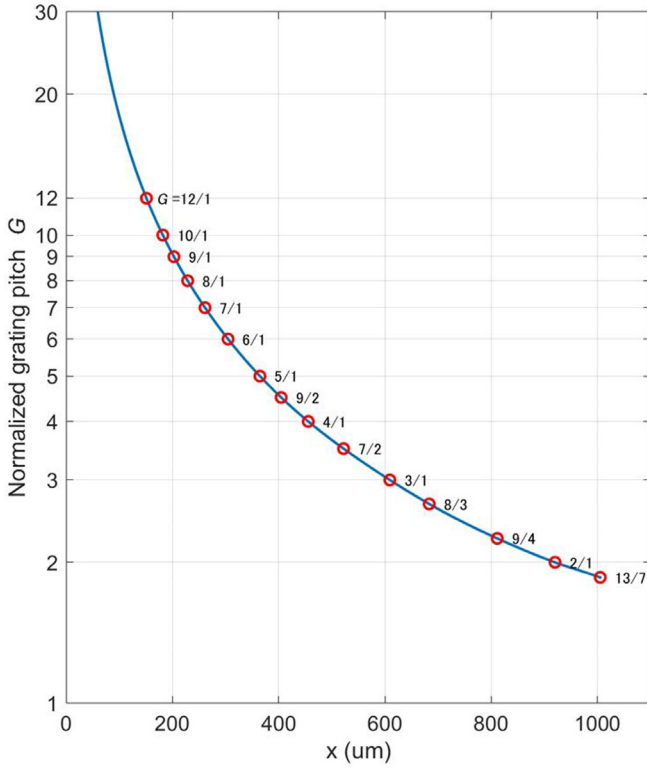


Fig. 9. Normalized grating pitch.

sections at 15 points identified in Figure 9. The simulations encompassed multiple incident angles selected from corresponding incident angle ranges, obtaining desired diffraction efficiency as the b -th order diffraction efficiency. It should be noted that in this RCWA simulation, we have modeled only the range from the glass substrate to the encapsulant. Consequently, the Fresnel reflection losses at the air interfaces of the glass substrate and encapsulant are not accounted for. By interpolating and extrapolating this diffraction efficiency data with respect to x -coordinates and incident angles, we generate the diffraction efficiency map as depicted in Figure 11. The efficiency data points used for generating this map are denoted by black circles in Figure 11. This map illustrates the dependence of diffraction efficiency on both position and angle. Subsequently, we evaluate the focusing efficiency for meridional rays (Fig. 12) by averaging the diffraction efficiency for multiple meridional rays reaching each image height at their specific x -coordinates and angles of incidence on the metasurface, as extracted from the diffraction efficiency map. For instance, the meridional rays that reach the image height ratio (image height normalized by the maximum image height) of 0.8 pass through the x -axis of the metasurface from 301 to 751 μm at an incident angle of $\theta = 30.2^\circ$. The focusing efficiency at this image height ratio is calculated by averaging the diffraction efficiencies at 100 equidistant points along the red line, as shown in

Figure 11. As previously mentioned, since the diffraction efficiency decreases toward the lens edge, the focusing efficiency also diminishes with higher image height.

3.2 Proposed design

Herein we describe a metalens designed by the proposed method, based on the same optical configuration as depicted in Figures 7 and 8. In the center region, within a radial distance of 455.6 μm , pillars are arranged in the same manner as the conventional design in the previous subsection (Fig. 13a). This boundary between the center and outer regions is selected as the point where the normalized grating pitch G , as shown in Figure 9, equals 4/1. In the outer region, the angular period is 0.0552° where the radial distance is from 455.6 to 848.9 μm . This choice of angular period is intended to ensure that the ratio of the y -directional width of the unit-cell to the circumferential width of the innermost supercell is 3:2 (Fig. 13b). Selecting these parameters in this manner facilitates the optimization and validation of phase continuity at the boundary between the center region and innermost supercell. By adjusting the angular period with a ratio of 2:1 (Fig. 13c), the angular period is 0.0276° where the radial distance is over 848.9 μm . This ensures that the circumferential width of each supercell is always below the operating wavelength, $\lambda = 940$ nm, as shown in Figure 14. By maintaining the circumferential width below λ , the unwanted light with orders in the circumferential direction is completely reflected at the interface between the encapsulant and the air. This prevents unwanted light from directly reaching the image plane. To prevent the generation of unwanted light within the encapsulant, it is advisable to maintain the circumferential width below the wavelength in encapsulant $\lambda_{\text{encapsulant}} = 940 \text{ nm}/n_{\text{encapsulant}} = 653$ nm. Even in regions where the circumferential width of the supercells exceeds $\lambda_{\text{encapsulant}}$, our design can still achieve a high diffraction efficiency approaching 90%, as observed in Figure 15. The benefit from adjusting the angular period with a ratio of 2:1 is described subsequently. Predefined parameters regarding the fabrication constraints are $s_{\text{min}} = 100$ nm, $l_{\text{min}} = 380$ nm, $d_{\text{min}} = 100$ nm, and $d_{\text{max}} = 280$ nm. The number of angle sampling for calculating the objective function is $N = 10$. Supercells at intervals of 10 are optimized. The objective function of each supercell after optimizations and the following inter/extrapolations is shown in Figure 15a. The pillar patterns in the several supercells where the optimizations from randomly generated initial patterns are performed are depicted in Figure 15b. At the 371st supercell, along with the adjustment of the angular period, the number of pillars in the supercell has decreased from 4 to 2. In addition, for the 391st, 411th, and 551st supercells, where the objective function exhibits a decline exceeding 5% as a result of optimization based on the metagrating design inherited from previous supercell, the optimization is reinitiated with randomly generated initial patterns. In Figure 15a, multiple pronounced dips are observed, where the objective function decreases by more than 5% compared to adjacent

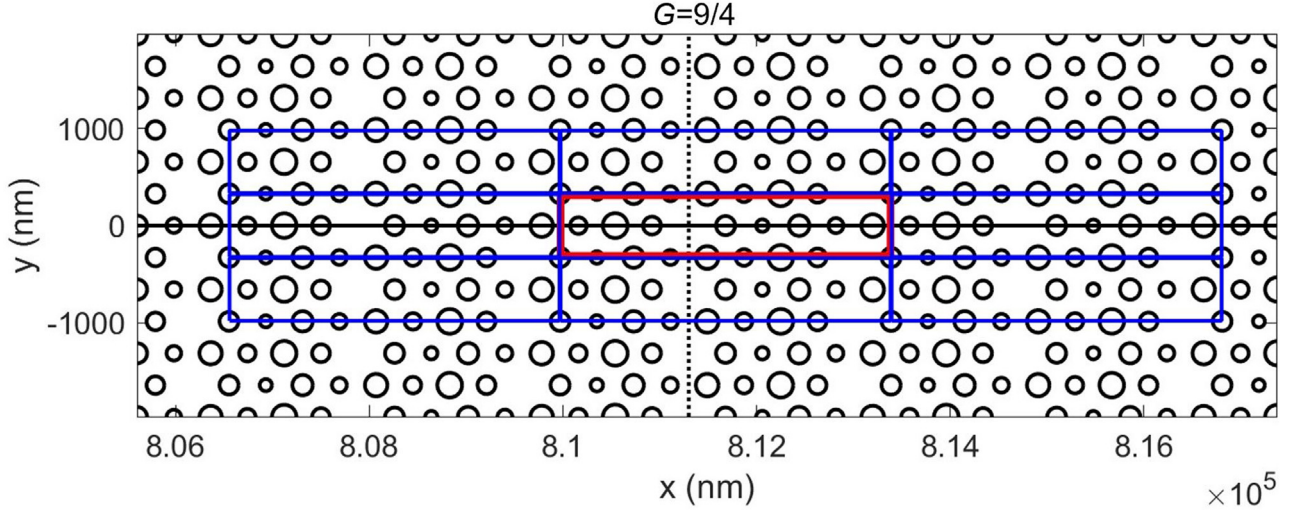


Fig. 10. Pillar arrangement near $x = 811\mu\text{m}$, where G equals $9/4$. The arrangement of pillars within the section including nine unit-cells (red frame) exhibits high similarity to that within the adjacent (blue frame).

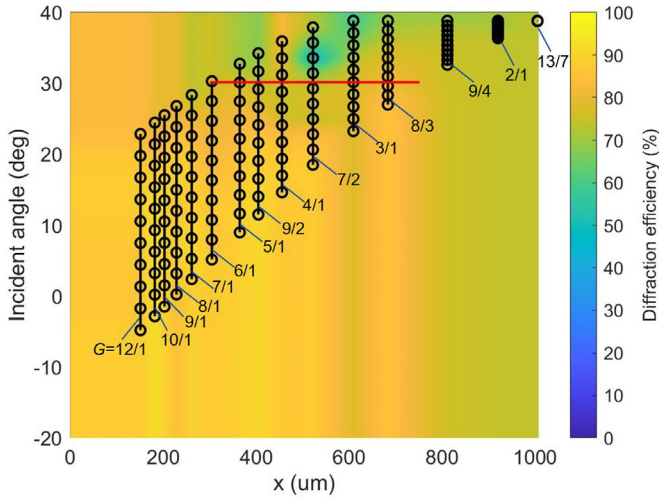


Fig. 11. Diffraction efficiency map of the conventional design.

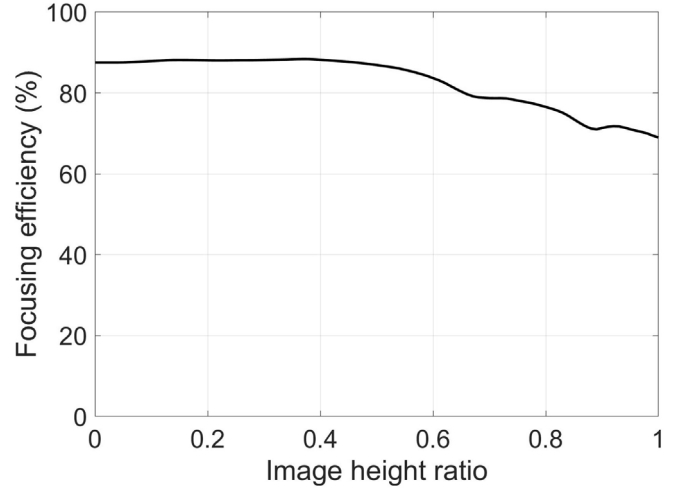


Fig. 12. Focusing efficiency of the conventional design for meridional rays.

supercells. These reductions occur in supercells, where the patterns are determined by inter/extrapolations. It is anticipated that improvements can be achieved by reducing the intervals at which selective optimizations are conducted.

In the previous section, we described our metagrating optimization process, which focused on maximizing angle- and polarization-averaged diffraction efficiency. This optimization stage, along with subsequent inter/extrapolations, do not incorporate considerations for phase continuities between adjacent supercells or at the boundary between center and outer regions. Thus, as a post-processing step, it becomes necessary to introduce a radial offset in the pillar arrangement within each supercell. While this offset does not guarantee perfect phase

continuity, it is a necessary adjustment to achieve as much phase continuity as possible. For the innermost supercell, due to the previously mentioned selection of the boundary, there exists a section (indicated by the green frame in Fig. 13b) near the boundary and just inside it, where the optical response can be approximately evaluated using RCWA (we subsequently refer to this section as “RCWA section in the center region.”). Therefore, we can optimize the offset for the innermost supercell using the RCWA results for the RCWA section in the center region and the innermost supercell. Here, when an offset γ is applied to the innermost supercell, the -1st order complex amplitude of the supercell for the TM and TE incident light with k -th incident angle are represented as $A_k^{\text{TM}} \exp(i\Phi_k^{\text{TM}}(\gamma))$ and $A_k^{\text{TE}} \exp(i\Phi_k^{\text{TE}}(\gamma))$, respectively.

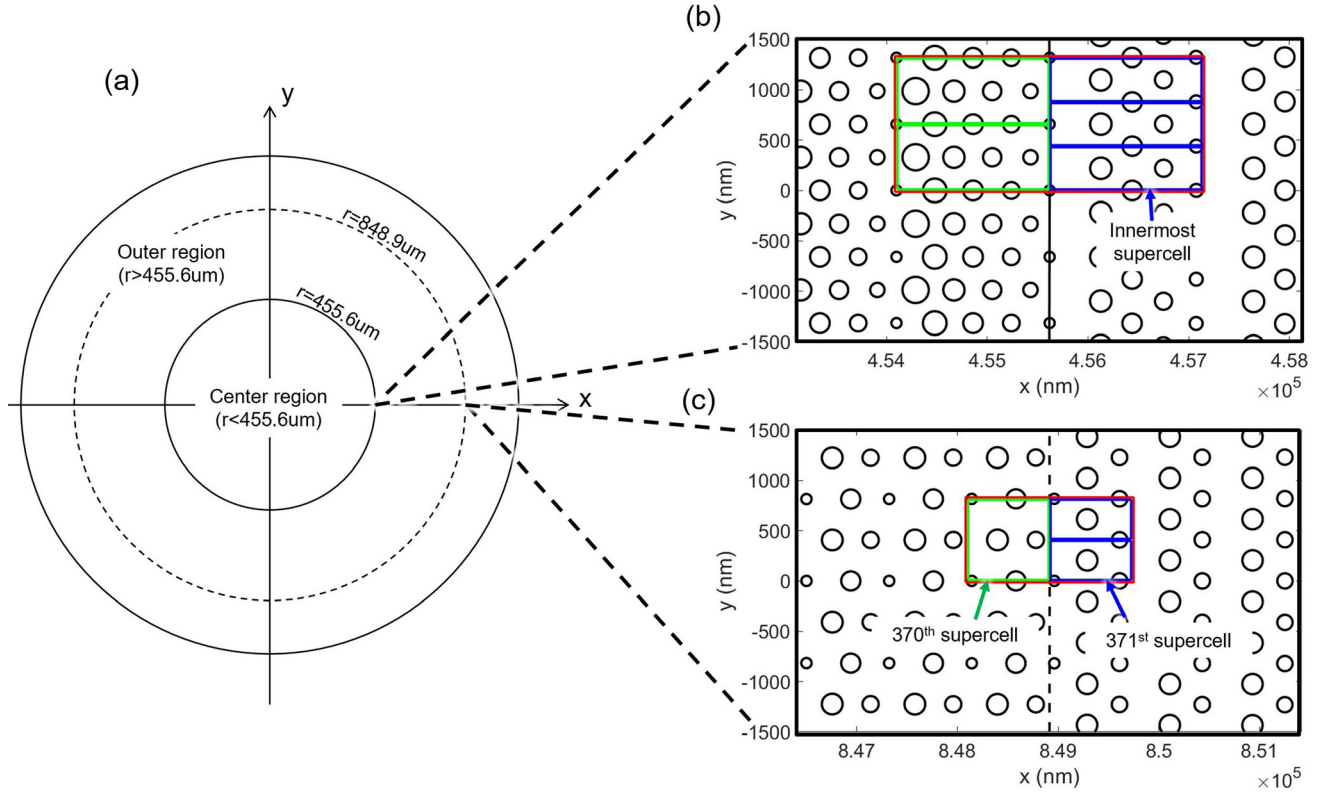


Fig. 13. (a) Overall top-view of the proposed design. (b) Pillar arrangement in the vicinity of the boundary between the center and outer region. (c) Pillar arrangement in the vicinity of the point where the angular period is adjusted to the smaller value.

Additionally, the -1st order complex amplitude of the RCWA section in the center region at that incident angle is denoted by $B_k^{\text{TM}} \exp(i\psi_k^{\text{TM}})$ and $B_k^{\text{TE}} \exp(i\psi_k^{\text{TE}})$, respectively. We define the phase mismatch $Q(\gamma)$ using the following expression:

$$Q(\gamma) = \frac{1}{2N} \sum_{k=1}^N [\exp(i\phi_k^{\text{TM}}(\gamma)) - \exp(i\psi_k^{\text{TM}})]^2 A_k^{\text{TM}} B_k^{\text{TM}} + [\exp(i\phi_k^{\text{TE}}(\gamma)) - \exp(i\psi_k^{\text{TE}})]^2 A_k^{\text{TE}} B_k^{\text{TE}}.$$

Under the fabrication constraints related to the spacing between pillars and the distance between their centers, we adopt the optimal offset γ that minimizes the phase mismatch $Q(\gamma)$. Notably, once RCWA is performed for a single offset γ_0 , the phase $\phi_k^{\text{TM}}(\gamma)$ and $\phi_k^{\text{TE}}(\gamma)$ for any offset γ is instantly calculated using the following expressions:

$$\phi_k^{\text{TM}}(\gamma) = \phi_k^{\text{TM}}(\gamma_0) + \frac{2\pi(\gamma - \gamma_0)}{L_x},$$

$$\phi_k^{\text{TE}}(\gamma) = \phi_k^{\text{TE}}(\gamma_0) + \frac{2\pi(\gamma - \gamma_0)}{L_x},$$

where L_x is the radial width of innermost supercell. Thus, there is no need for computations involving additional RCWA. Figure 16 displays the result of offset optimization for the innermost supercell. In Figure 16a, the solid line depicts the phase mismatch $Q(\gamma)$, computed for the same

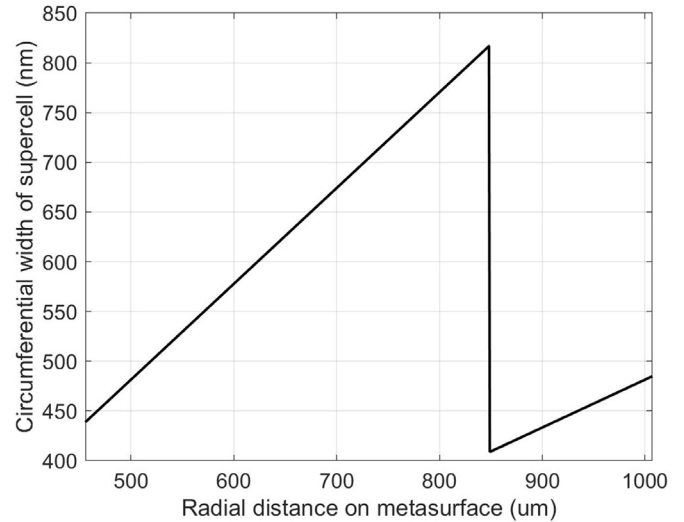


Fig. 14. Circumferential width of each supercell.

ten incident angles as those used in the metagrating optimization. Here, the origin of offset ($\gamma = 0$ nm) is defined to represent the situation where the pillars in the innermost supercell are positioned as close to the center region as possible under fabrication constraints. We find optimal offset to be around $\gamma = 120$ nm. Pillar arrangement with this optimal offset is depicted in Figure 16b. To validate the

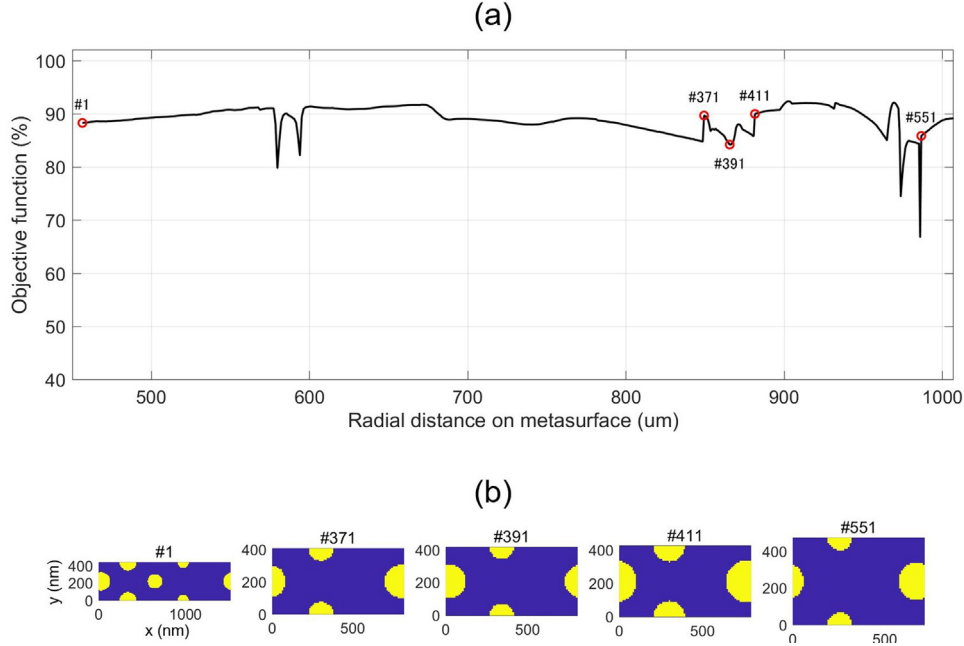


Fig. 15. (a) Objective function of each supercell after optimizations and the following inter/extrapolations. (b) Pillar patterns in the several supercells where the optimizations from randomly generated initial patterns are performed.

offset optimization based on the phase mismatch $Q(\gamma)$, we also perform RCWA for the area combining the RCWA section in the center region and the innermost supercell. Owing to a 3:2 ratio between the y -directional width of the unit-cell to the circumferential width of the innermost supercell, this combined area can be defined to include two RCWA sections in the center region and three innermost supercells, as depicted by the red frame in Figure 13b. The dependency of angle- and polarization-averaged desired (-2nd) order diffraction efficiency for this combined area on the offset is depicted by the dashed line in Figure 16a. The peak of this curve agrees well with the optimal offset with respect to the phase mismatch $Q(\gamma)$. Diffraction efficiencies for the combined area with optimal offset, RCWA section in the center region, and innermost supercell are shown in Figures 17a, 17b and 17c, respectively. Their respective averaged efficiencies across angles and polarizations are 84.4%, 81.0%, and 88.3%. The closeness of the first value to the average of the latter two (84.7%) indicates that the chosen optimal offset may effectively result in favorable phase continuity. For the supercells located further outward, we define the phase mismatch with the adjacent inner supercell in the same manner and determine an offset that minimizes this mismatch. This offset optimization is particularly important for the supercell where the angular period is reset, as the arrangement of pillars changes significantly. To assess the phase continuity after the offset optimization at this point, we run the RCWA for the area that combines supercells immediately inside and outside the angular period reset location (the 370th and 371st supercells). Since the ratio between the circumferential widths of these two supercells is 2:1, this combined area includes one 370th supercell and two 371st supercells, as illustrated by the red frame in Figure 13c. Diffraction efficiencies for the combined area with optimal offset, 370th

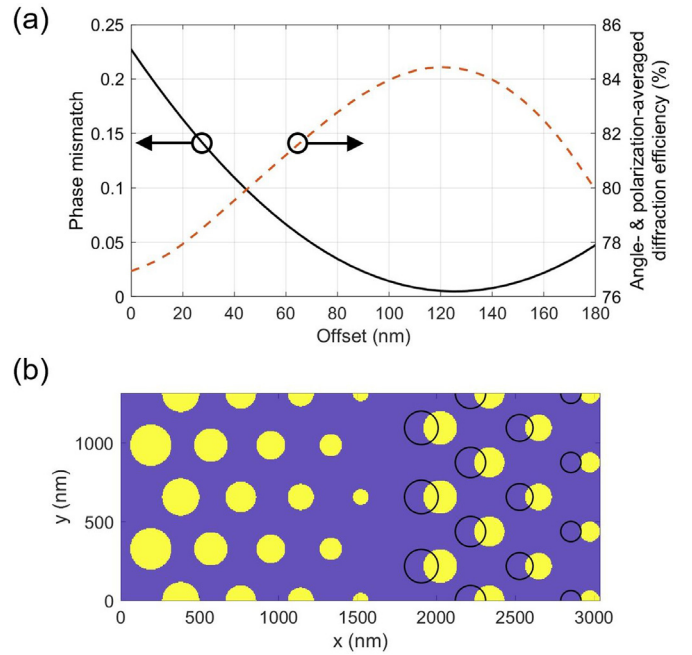


Fig. 16. Result of offset optimization for the innermost supercell. (a) Solid line depicts the phase mismatch Q . The dashed line depicts the dependency of angle- and polarization-averaged desired (-2nd) order diffraction efficiency for the combined area on the offset. (b) Pillar arrangement with the optimal offset with respect to the phase mismatch. Black circles depict the pillar arrangement with the offset $\gamma = 0$ nm.

supercell and 371st supercell are shown in Figures 18a, 18b, and 18c, respectively. Their respective averaged efficiencies across angles and polarizations are 83.0%, 84.8%, and 89.7%. Since the first value is lower than the latter two, the

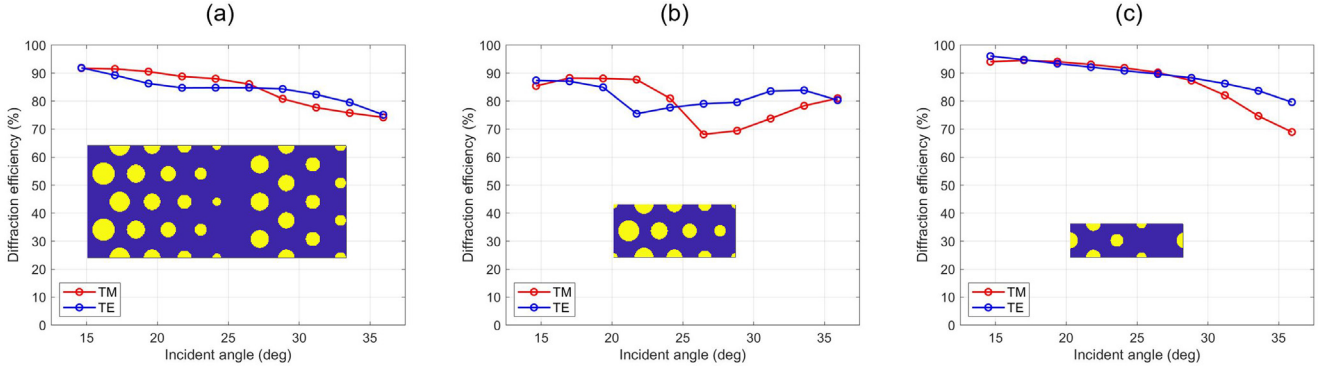


Fig. 17. (a) Diffraction efficiencies for the combined area with the optimal offset. (b) Diffraction efficiencies for the RCWA section in the center region. (c) Diffraction efficiencies for the innermost supercell.

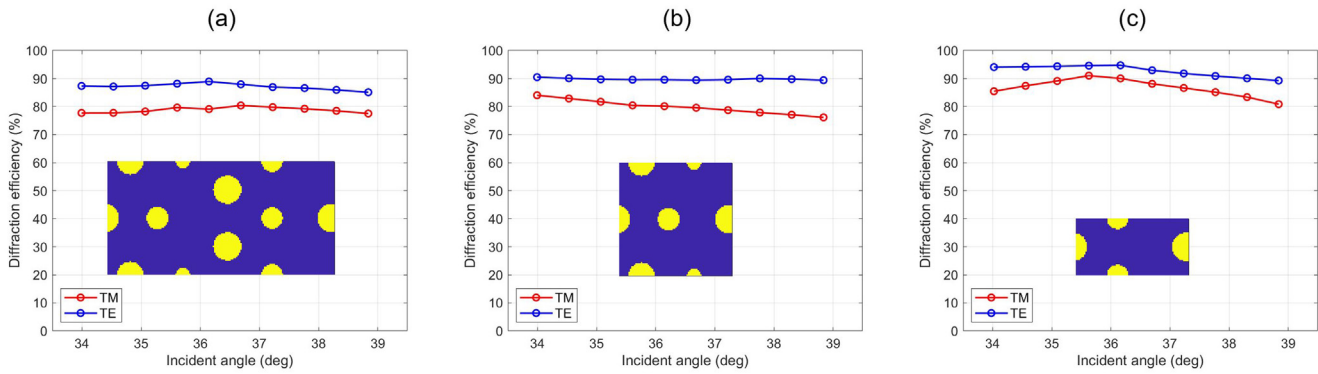


Fig. 18. (a) Diffraction efficiencies for the combined area with optimal offset. (b) Diffraction efficiencies for the 370th supercell. (c) Diffraction efficiencies for the 371st supercell.

incident light on this combined area might generate a few percent of undesired scattered light due to phase inconsistencies not fully resolved by offset optimization. However, this value is significantly higher compared to the averaged efficiency across same angles and polarizations of the conventional design at this location (75.0%), as evaluated using the efficiency map in Figure 11. This suggests the superiority of our proposed design over the conventional design even in this area.

For this proposed design, we evaluated the focusing efficiency for meridional rays in the same manner as applied to the conventional design in the previous subsection. The diffraction efficiency map for the proposed design is presented in Figure 19. Because the conventional method was adopted in the center region, the diffraction efficiency map for this range ($x < 455.6 \mu\text{m}$) is the same as that in Figure 11. In the outer region, the values for this range of the diffraction efficiency map were obtained using RCWA results for all supercells. The focusing efficiency for meridional rays calculated using this diffraction efficiency map is displayed in Figure 20. In the area with an image height ratio up to 0.40, the rays pass solely through the center region. Therefore, there is no difference between the focusing efficiencies of the conventional and proposed methods in this region. When the image height ratio is higher, resulting in a greater amount of light being

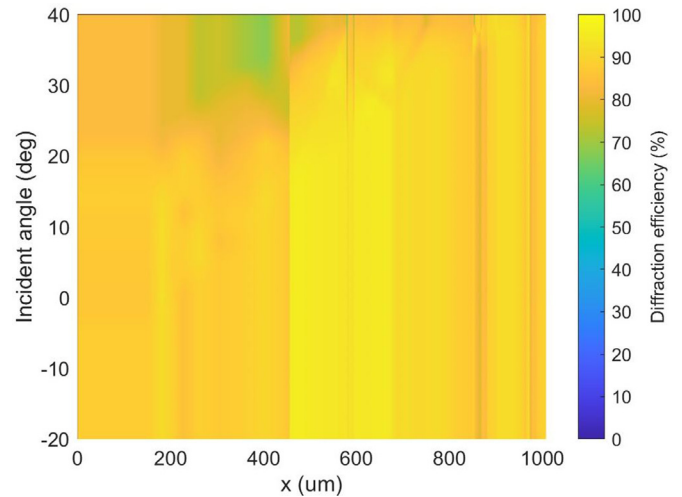


Fig. 19. Diffraction efficiency map of the proposed design.

diffracted by optimized metagratings in the outer region, the contrast in focusing efficiencies becomes more evident. The proposed method achieves an efficiency of over 85% for all image heights, whereas the efficiency of the conventional method drops to 70% at the edge.

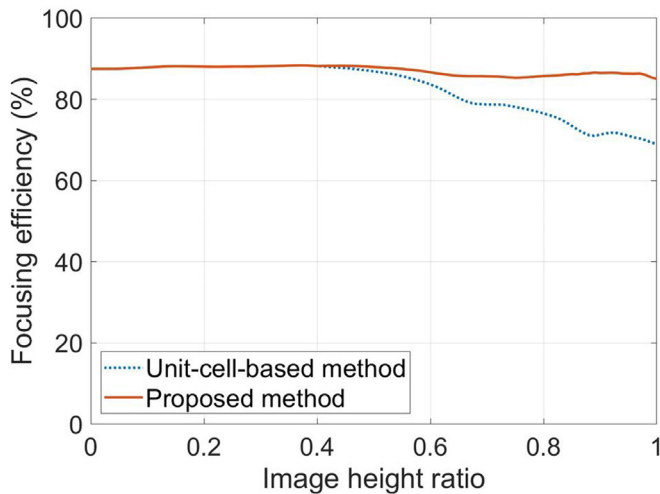


Fig. 20. Focusing efficiency of the proposed design for meridional rays.

4 Conclusion

This study proposed a novel design method for wide field-of-view monochromatic metalenses to address the design challenges in achieving high efficiency and computational scalability. The entire metalens was divided into the center and outer regions. In the center region, the unit-cell-based method was applied. The outer region was subdivided into supercells with their sizes comparable to the wavelength. The pillar pattern in each supercell was determined through metagrating optimization using the adjoint technique. The proposed method significantly improved the simulated focusing efficiency for meridional rays of the high image height region. This advantage stems from the optimized pillar arrangement in the outer of the lens, which accounts for the inter-pillar couplings and the range of incidence angles. The proposed method makes the design of large-scale wide field-of-view high-efficiency monochromatic metalenses more tractable than the existing design methods, potentially paving the way for the widespread adoption of monochromatic metalenses in commercial sensors in the future.

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Conflicts of interest

The authors have nothing to disclose.

Data availability statement

Data associated with this article cannot be disclosed due to company information management regulations.

Author contribution statement

Conceptualization, H.T. and T.Y.; Methodology, H.T.; Software, H.T.; Validation, H.T.; Formal Analysis, H.T.; Writing – Original Draft Preparation, H.T.; Writing – Review & Editing, H.T. and T.Y.; Visualization, H.T.; Supervision, T.Y..

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