Theoretical study on dynamical planar-chirality switching in checkerboard-like metasurfaces

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Abstract – In this paper, we show that the handedness of a planar chiral checkerboard-like metasurface can be dynamically switched by modulating the local sheet impedance of the metasurface structure. We propose a metasurface design to realize the handedness switching and theoretically analyze its electromagnetic characteristic based on Babinet’s principle. Numerical simulations of the proposed metasurface are performed to validate the theoretical analysis. It is demonstrated that the polarity of asymmetric transmission for circularly polarized waves, which is determined by the planar chirality of the metasurface, is inverted by switching the sheet impedance at the interconnection points of the checkerboard-like structure. The physical origin of the asymmetric transmission is also discussed in terms of the surface current and charge distributions on the metasurface.

Key words: Planar chirality, Asymmetric transmission, Tunable metamaterials, Babinet's principle.

1 Introduction

Chiral structures have been one of the most interesting subjects in metamaterials research [1, 2]. In analogy to the chirality of three-dimensional structures, a planar structure is said to be planar chiral if it has no line of mirror symmetry in its structure plane (e.g., an Archimedean spiral) [3, 4]. In other words, a planar chiral structure cannot be superimposed on its in-plane mirror image by in-plane rotations and/or translations. Note that the handedness of a planar chiral structure is reversed if it is observed from the opposite side of its structure plane. Polarization effects related to the planar chiral structures for electromagnetic waves have been studied in terms of optical activity and reciprocity for scattered light [5–7]. In addition, it has been identified that the total transmission intensity of circularly polarized waves normally incident on planar chiral structures depends on the direction of the wave propagation, provided that the planar chiral structures are anisotropic and lossy [8]. This phenomenon is called asymmetric transmission for circularly polarized waves and is consistent with the Lorentz reciprocity theorem, although it is seemingly nonreciprocal. Asymmetric transmission of planar chiral metamaterials has been experimentally reported in from microwave [8, 9] and terahertz [10] to optical frequency regions [11]. Moreover, it has also been extended to linearly polarized waves [12].

Recently, metamaterials and metasurfaces with dynamically tunable electromagnetic properties have been extensively studied towards applications to the advanced control of electromagnetic wave propagation [13, 14]. The recent progress has enabled the dynamical tuning of optical activity resulting from the chirality of artificial structures by employing photoexcitation of semiconductors [15–19], micro-electro-mechanical systems [20, 21], and phase change materials [22]. On the other hand, the possibility of dynamical tuning of planar chirality and the associated asymmetric transmission for circular polarizations remains unexplored in literature.

In this paper, in order to realize dynamical planar-chirality switching, we utilize the singular characteristic of metallic checkerboard-like structures, which is critically dependent on the electrical connectivity at the interconnection points of their metallic patches [23–32]. We have shown that their characteristics can be understood in terms of Babinet’s principle, which is extended to screens with finite sheet impedances [30, 31]. According to Babinet’s principle, by switching the local sheet impedance at the interconnection points, the responses of the checkerboard-like structures can dynamically alternate between two contrasting behaviors, such as capacitive-inductive responses [33] and anisotropic transmission characteristics for orthogonal linear polarizations [34]. Here, we apply such dynamical-checkerboard designs to dynamical planar-chirality switching. In contrast to the previous designs [33, 34], a new design contains resonant planar chiral structures, which enable planar-chirality switching of the whole structure.
This paper is organized as follows. First, in Section 2, we present the design of a metasurface that enables dynamical planar-chirality switching, and theoretically demonstrate its capability of planar-chirality switching based on Babinet’s principle. In Section 3, we show the results of numerical simulations of the presented metasurface using the finite element method and discuss the physical origin of planar-chirality switching. Finally, the conclusions are drawn in Section 4.

2 Design and operation principle

2.1 Babinet’s principle for the incidence of circularly polarized waves

Before proceeding to a theoretical analysis of planar chiral metasurfaces based on Babinet’s principle, we review Babinet’s principle in particular for the incidence of circular polarization. Babinet’s principle relates scattering problems of metasurfaces to those of their complementary structures. We assume that a metasurface in a vacuum has a spatially varying sheet impedance of \( Z(x, y) \), where \((x, y)\) is the coordinate on the metasurface. Then, the sheet impedance of its complementary metasurface is given by

\[
Z_s^{(c)}(x, y) = \frac{Z_0^2}{4Z(x, y)},
\]

where \( Z_0 \) is the impedance of a vacuum. This transformation is called the impedance inversion [30]. The electromagnetic fields in the two problems are related to each other if the incident fields in the complementary scattering problem satisfy \( (E_s^{(c)}, H_s^{(c)}) = (\pm Z_0 H_{\text{in}}, \mp E_{\text{in}}/Z_0) \), where \((E_{\text{in}}, H_{\text{in}})\) is the incident electromagnetic field in the original problem. In particular, for the incidence of linearly polarized plane waves, this interchange of \( E \) and \( H \) is equivalent to the polarization rotation by \( \pi/2 \). On the other hand, for the incidence of circularly polarized plane waves, the incident polarization state in the complementary problem is equal to that in the original problem, up to a phase factor of \( \exp(\pm i\pi/2) \), which depends on the handedness of the incident circular polarization [30]. For example, let us consider a problem shown in Figure 1a. Its complementary problem is depicted in Figure 1b. For the incidence of left circularly polarized (LCP) plane waves, we can derive the following equations via Babinet’s principle [30]:

\[
\tilde{t}_{LL} + \tilde{t}_{LL}^{(c)} = 1,
\]

\[
\tilde{t}_{RL} - \tilde{t}_{RL}^{(c)} = 0,
\]

where \( \tilde{t}_{ij} \) and \( \tilde{t}_{ij}^{(c)} \) (\( i, j = L, R \)) denote the complex amplitude transmission coefficients from incident \( j \) circular polarization to \( i \) circular polarization in the original and complementary problems, respectively. For the incidence of right circularly polarized (RCP) waves, L and R in equations (2) and (3) are interchanged.

2.2 Design of the metasurface for planar chirality switching

Let us consider a planar chiral checkerboard-like metasurface in a vacuum, as shown in Figure 2a. We assume it to be infinitely periodic and infinitely thin. At the interconnection points of the perfect-electric-conductor (PEC) patches, there are variable impedance sheets with a sheet impedance of \( Z_s \). Deformed gammadiodes (hereafter, RT’s) [35, 36] and their complementary in-plane mirror images (CLT’s) are embedded in the holes and the PEC patches of a plain checkerboard-like structure so that the metasurface with \( Z_s = Z_0/2 \) is congruent with the in-plane mirror image of its complementary structure. The handedness of the planar chiral metasurface is determined by the handedness of RT’s and CLT’s, and by the electrical connectivity of the PEC patches. As mentioned above, electromagnetic responses of metallic checkerboard-like structures critically depend on the electrical connectivity at the interconnection points of their PEC patches. Here, we initially assume \( Z_s = Z_s^{(\text{high})} \gg Z_0/2 \). In other words, the PEC patches are virtually disconnected.

To evaluate the asymmetric transmission of the metasurface, we define the total-transmission-intensity difference \( \Delta T_L \) [8] for the LCP waves by

\[
\Delta T_L := (T_{LL} + T_{RL}) - (T_{LL} + T_{RL}) = T_{RL} - T_{RL},
\]

where power transmittance \( T_{ij} := |\tilde{t}_{ij}|^2 \) and the symbol \( \rightarrow \) (\( \rightarrow \)) indicate normal incidence from \( z > 0 \) (\( z < 0 \)) here. We assume that scattering into diffraction modes is negligible. In other words, the periodicity of the metasurface is smaller than the incident wavelength. Note that \( T_{LL} = -T_{RL} \) is satisfied owing to the reciprocity. From the mirror symmetry with respect to the \( z = 0 \) plane, we can show \( \Delta T'_R = -\Delta T_L \) for the normal incidence of the RCP waves. Thus, we will omit the subscript as \( \Delta T := \Delta T'_L \) for simplicity below. The sign of \( \Delta T \) is determined by the handedness of the planar chiral metasurface observed from \( z > 0 \). We note that \( \Delta T \) can also be written as \( \Delta T = T_{RL} - T_{LR} \) because we have \( T_{RL} = T_{LR} \) from the reciprocity. Thus, the asymmetric transmission for circularly polarized waves can
be regarded as dichroism with respect to circular polarization conversion.

2.3 Theoretical analysis of planar-chirality switching based on Babinet’s principle

Figure 2b shows the complementary structure of Figure 2a, the electromagnetic responses of which are related through Babinet’s principle to those of the original one. In the complementary structure, the original PEC (hole) areas are changed to holes (PEC), and the impedance sheets with finite sheet impedance $Z_{b}^{(hi)}$ are transformed into impedance sheets with $Z_{b}^{(lo)} = Z_{b}^{(hi)} / (4Z_{b}^{(hi)})$ via equation (1). According to Babinet’s principle, equation (3), the polarization conversion efficiency of the complementary structure is equal to that of the original one, i.e., $T_{RL}^{(c)} = T_{RL}^{(hi)}$ and $P_{RL}^{(c)} = P_{RL}^{(hi)}$ hold, where the superscripts “(c)” and “(hi)” represent quantities of the complementary and high-impedance structures, respectively. Therefore, we have

$$\Delta T^{(c)} = \Delta T^{(hi)}.$$  

On the other hand, by switching the sheet impedance at the interconnection points from $Z_{b}^{(hi)}$ to $Z_{b}^{(lo)} \ll Z_{0}/2$, the metasurface shown in Figure 2a changes to the structure shown in Figure 2c. If $Z_{b}^{(lo)} = Z_{0}^{2} / (4Z_{b}^{(hi)}$) is satisfied, the low-impedance structure shown in Figure 2c is congruent with the in-plane mirror image of the complementary structure shown in Figure 2b. In other words, Figure 2c is the observation of Figure 2b from the opposite $(z < 0)$ side. Thus, we have

$$T_{RL}^{(lo)} = T_{RL}^{(hi)}$$

where equation (5) is used. This equation indicates that the sign of $\Delta T$ is inverted just by switching the sheet impedance at the corners. Therefore, the handedness of the planar chiral metasurface is dynamically switchable by using variable impedance sheets. Note that we have

$$\Delta T^{(lo)} = -\Delta T^{(hi)} = 0$$

in the limit of $Z_{b}^{(hi)} = \infty$ and $Z_{b}^{(lo)} = 0$, where the metasurface is lossless. This is because loss is essential for the asymmetric transmission in planar chiral structures [8]. We also note that

$$\Delta T^{(c)} = -\Delta T^{(hi)} = 0$$

if $Z_{b}^{(hi)} = Z_{b}^{(lo)} = Z_{0}/2$.

3 Numerical simulation

3.1 Planar chiral checkerboard-like metasurface in a vacuum

In order to validate the theoretical analysis in Section 2, we numerically simulated the electromagnetic responses of the proposed metasurface using the finite element method solver (ANSYS HFSS). Figure 3a shows the unit cell of the simulated metasurface, which was placed at $z = 0$ in the computational domain of a vacuum. The geometrical parameters in Figure 3a are: $a = 300 \mu m$, $d = 30 \mu m$, $l = 180 \mu m$, $p = 80 \mu m$, and $w = 10 \mu m$. The CLs have the same parameters as the RIs. There are sheets with variable sheet impedances $Z_{s}$ at the interconnection points of the PEC patches.
We imposed periodic boundary conditions on the side boundaries of the computational domain to simulate an infinitely periodic system. In the simulation, the complex amplitude transmission coefficients for the normal incidence of $x$ and $y$ linearly polarized waves were calculated, and then, the transmission coefficients were transformed into those in the circular-polarization basis.

The calculated $\Delta T$ spectra for $Z_s/Z_0 = 25$ and 0.01 are shown in Figure 3b. Note that $Z_s/Z_0 = 25$ and 0.01 are complementary to each other via the impedance inversion. We observe $|\Delta T|$ is resonantly enhanced around 0.504 THz. In addition, it is clearly confirmed that the polarity of the transmission asymmetry is inverted by switching the sheet impedance $Z_s$, which agrees well with equation (6). In other words, the handedness of the planar chiral metasurface is dynamically switchable by modulating its local sheet impedance $Z_s$.

To understand the resonant character of $\Delta T$ around 0.504 THz, we simulated the distributions of the surface current $J_{\text{surf}}$ on the metasurface for the incidence of the LCP waves from the front ($z > 0$) or back ($z < 0$) sides of the metasurface structure at 0.504 THz: (a) $Z_s/Z_0 = 25$ with front-side incidence; (b) $Z_s/Z_0 = 25$ with back-side incidence; (c) $Z_s/Z_0 = 0.01$ with front-side incidence; (d) $Z_s/Z_0 = 0.01$ with back-side incidence. The color maps and arrows in green indicate $D_z$ and the surface current vector, respectively.

Figure 4. Distributions of the electric flux density $D_z$ at $z = 1 \mu$m and surface current $J_{\text{surf}}$ on the metasurface for the incidence of the LCP waves from the front ($z > 0$) or back ($z < 0$) sides of the metasurface structure at 0.504 THz: (a) $Z_s/Z_0 = 25$ with front-side incidence; (b) $Z_s/Z_0 = 25$ with back-side incidence; (c) $Z_s/Z_0 = 0.01$ with front-side incidence; (d) $Z_s/Z_0 = 0.01$ with back-side incidence. The color maps and arrows in green indicate $D_z$ and the surface current vector, respectively. For clarity, the phase of $D_z$ is shifted by $\pi/2$ relative to $J_{\text{surf}}$.

In the case of $Z_s/Z_0 = 25$ with front-side incidence of the LCP waves shown in Figure 4a, a dipolar current along the $y$ direction is induced on $R_C$, which causes a local magnetic field $H_{\text{loc}}^z(z = +0) = -H_{\text{loc}}^z(z = -0)$ in the $x$ direction around the bars of $R_C$. Generally, such a dipolar current contributes to circular polarization conversion because its $y$-polarized emission is a superposition of the LCP and RCP waves [9]. However, the calculated $T_{RL}$ spectrum shown in Figure 3c shows a dip around 0.504 THz. Therefore, the emission from the resonance mode in $R_C$’s destructively interferes with that from the charge accumulated at the corners of the
checkerboard structure (see Appendix). As mentioned above, the case of $\frac{Z_{d}}{Z_{0}} = 0.01$ with back-side incidence shown in Figure 4d is the complementary problem of the case shown in Figure 4a. In Figure 4d, we confirm a loop current induced around the arms of CLTs and charge accumulation along the slits of CLTs in the $y$ direction, which causes a local electric field $E_{\text{loc}}$ in the $x$ direction in the slits of CLTs. This resonance mode characterized by $E_{\text{loc}}$ is complementary to the one characterized by $H_{\text{loc}}$ in Figure 4a. We note that the polarization of the waves emitted from the complementary mode of CLTs is rotated by $\pi/2$ compared to that from the RTs due to the interchange of $E$ and $H$ in Babinet’s principle. The dip in the $T_{\text{RL}}$ spectrum can be explained in the same way as above.

On the other hand, in the cases of $\frac{Z_{d}}{Z_{0}} = 25$ with back-side incidence (Figure 4b) and $\frac{Z_{d}}{Z_{0}} = 0.01$ with front-side incidence (Figure 4c), both the resonance modes of RTs and CLTs are excited. The phase maximizing $E_{\text{loc}}$ in CLTs is delayed by approximately $\pi/2$, relative to that maximizing $H_{\text{loc}}$ in RTs. Here, note that $D_z$ is depicted with a $\pi/2$ phase shift relative to $J_{\text{surf}}$ in Figure 4. Hence, the RCP components are dominantly emitted from RTs and CLTs in the $x$ directions of Figures 4b and 4c, and the corresponding $T_{\text{RL}}$ exhibits a relatively large value around 0.504 THz, as shown in Figure 3c.

From an application perspective, continuous tunability of $\Delta T$ is also a desired feature. Figure 5 shows the calculated $Z_a$ dependence of $\Delta T$ at 0.504 THz. It is confirmed that $\Delta T$ can be continuously tuned by controlling $Z_a$ around $Z_0/2$. As predicted in Section 2, $\Delta T$ tends to zero when $Z_a \rightarrow \infty$ or 0 due to the disappearance of lossy elements. $\Delta T$ is also nearly zero at $Z_a = Z_0/2$. The maximum value of $|\Delta T|$ is about 0.22 in the present case. There is little room for further increasing $|\Delta T|$ by optimizing the geometry of the deformed gammadions as $T_{\text{RL}}$ is theoretically bound by 0.25 for electric single-layer structures [37].

### 3.2 Planar chiral checkerboard-like metasurface on a dielectric substrate

To actually materialize the proposed metasurface, there must be a dielectric substrate supporting the thin metasurface structure. The existence of the substrate breaks the mirror symmetry with respect to $z = 0$ and thereby invalidates Babinet’s principle and equation (6). This difficulty can be approximately circumvented by just sandwiching the metasurface structure between the same dielectric plates [31]. In addition, it is reasonable to expect that similar responses are obtained if the metasurface is placed on a dielectric substrate with a moderate refractive index. Here, we will numerically demonstrate that sign inversion of $\Delta T$ can occur for a metasurface placed on a dielectric substrate. We simulated a similar model to Figure 3a with the computational domain in $z < 0$ filled by a lossless dielectric material having a refractive index of $n = 3.4$, which corresponds to that of high-resistivity silicon in the terahertz regime [38]. To reflect the wavelength-shortening effect in the substrate, all the geometrical parameters were divided by $n$. Figure 6 shows the calculated $\Delta T$ spectra for $Z_a/Z_{0} = 25$ and 0.01, where $Z_a := Z_0/n$. We can confirm a sign inversion of $\Delta T$, although the spectral shapes of $|\Delta T|$ are not identical between the two cases. This is attributed to the broken mirror symmetry with respect to $z = 0$ caused by the substrate.

### 4 Conclusion

In conclusion, we investigated the dynamical handedness switching in a planar chiral checkerboard-like metasurface loaded with switchable impedance sheets. We theoretically and numerically showed that the polarity of the asymmetric transmission in the proposed planar chiral metasurface can be inverted by modulating the sheet impedance at the interconnection points of the checkerboard-like structure. Such impedance switching can be experimentally implemented in the terahertz regime by exploiting, for example, photoexcitation of semiconductors [16, 17, 39] and metal-insulator transitions of vanadium dioxide [33, 34]. We explained the behaviors of the metasurface from the microscopic point of view, i.e., the field distributions. Note that we can also discuss them in terms of macroscopic parameters such as averaged sheet admittance tensors [40, 41], which facilitate design of metasurfaces with desired transmission and reflection.
characteristics. Finally, it should be noted that although the simulation was performed in the terahertz regime, the theory discussed in this paper is also applicable to the other frequency ranges, where metals exhibit high conductivity and, thus, Babinet’s principle is valid.

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References

Appendix

Origin of the dip in the cross-polarized transmission spectra

To understand the origin of the dip around 0.504 THz in the \( T_{RL} \) spectra, we calculated \( T_{RL} \) spectra of the metasurface without gammadions of one handedness. Figure 7 shows the \( T_{RL} \) spectra of the metasurface with and without RGs for \( Z_s/Z_0 = 25 \). While the spectrum is relatively flat in the case without RGs, the case with RGs shows an acute spectral change around 0.5 THz. In addition, we can confirm the enhancement of conversion around 0.46 THz when RGs are added to the structure. These observations can be understood as in the case of Fano resonance [42] by considering constructive and destructive interference between the resonance of RGs and the broader background spectrum of the checkerboard metasurface without RGs, which is caused by the charge accumulated at the corners of the checkerboard structure.

Figure 7. Calculated \( T_{RL} \) spectra of the metasurface with and without RGs for \( Z_s/Z_0 = 25 \).