Breaking the black-body limit with resonant surfaces

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Abstract – The speed with which electromagnetic energy can be wirelessly transferred from a source to the user is a crucial indicator for the performance of a large number of electronic and photonic devices. We expect that energy transfer can be enhanced using special materials. In this paper, we determine the constituent parameters of a medium which can support theoretically infinite energy concentration close to its boundary; such a material combines properties of Perfectly Matched Layers (PML) and Double-Negative (DNG) media. It realizes conjugate matching with free space for every possible mode including, most importantly, all evanescent modes; we call this medium Conjugate Matched Layer (CML). Sources located outside such layer deliver power to the conjugate-matched body exceptionally effectively, impressively overcoming the black-body absorption limit which takes into account only propagating waves. We also expand this near-field concept related to the infinitely fast absorption of energy along the air-medium interface to enhance the far-field radiation. This becomes possible with the use of small particles randomly placed along the boundary; the induced currents due to the extremely high-amplitude resonating fields can play the role of emission “vessels”, by sending part of the theoretically unlimited near-field energy far away from the CML structure.

Key words: Black-body limit, Conjugate matching, Perfectly Matched Layer (PML), Wireless power transfer.

1 General principles

1.1 Introduction and motivation

The selection of configurations and materials to achieve a desired distribution of electromagnetic energy in space and time is a general and fascinating topic covering a broad scientific area within electromagnetics and photonics. In particular, how the available energy from a primary source can be very rapidly and suitably delivered to another region of space is an issue with numerous applications. Designs of effective photovoltaic cells [1, 2], wireless charging components [3, 4] and light steering subwavelength metasurfaces [5, 6] are only a few of the application areas with huge practical interest.

Additionally, one remarkable class of problems where wireless transfer of power and its distribution is crucial concerns understanding, design, and fabrication of electromagnetic absorbers. A thorough overview of potential topologies for ultra-efficient absorbers wrapped into a general theory of thin perfect absorbing layers is provided in [7], where suitable models and designs are given for each of the identified families. Furthermore, so-called perfect metamaterial absorbers have been fabricated with sole use of metallic elements which are experimentally tested with excellent results [8], and are examined in alternative structures [9, 10]. Other interesting designs in the visible spectrum [11, 12], the sub-THz spectrum [13] and at radio frequencies [14] have been reported to exhibit high efficiency combined with broadband features.

It should be stressed that the performance of all the aforementioned absorbing configurations is bounded by the performance of the so-called ideal black body, which completely absorbs all the incident electromagnetic radiation, regardless of the angle of incidence and polarization; the corresponding concept is widely used in thermodynamics, optics, and radio engineering. Several attempts to emulate the response of a black body have been made in acoustics [15] and photonics [16] with some success. However, very recently absorbing structures which break that upper limit posed by perfectly black body have been proposed [17, 18], and they are based on the use of Double-Negative (DNG) uniaxial media which obey the Perfectly Matched Layer (PML) rule. That Conjugate Matched Layer (CML), as we call it, and its variants, would be the major topic of the present work, where alternative excitations are considered. In particular, the proposed CML concept is described in the Section 1.2, while its enormous, super-Planckian absorbing performance is demonstrated in Section 2. The ability of the analyzed component to send a part of the giant near-field power concentrated at its surface, to the far-zone, is examined in Section 3. In particular, we perturb slightly the structure by putting one (Sect. 3.1)
or multiple (Sect. 3.2) particles in the region with strong background field. Due to the diffuse scattering occurred from the induced currents, the far-field radiation of the device becomes stronger than that of the corresponding ideal black body.

1.2 Conjugate matched layer (CML) concept

In order to maximize the power wirelessly delivered from a source to a load, the load should be conjugate matched to the internal impedance of the source. This well-known maximal power principle applied in circuits can be generalized to cover electrically sizable electromagnetic structures. The only difference is that, in the latter case, there are infinitely many channels (modes) for transferring energy; if all of them obey the conjugate-matching principle, the transferred power is diverging [17]. In particular, the load can be replaced by a semi-infinite half-space filled with a uniaxial medium of relative constituent properties \( (\varepsilon_r, \mu_r, \varepsilon_n) \) and the source by any dipole or multipole placed in the vicinity of the air-medium interface [18]. Considering TM (magnetic field with one sole Cartesian component) illumination (which does not damage the generality), the internal impedance of the source is the one of free space: 

\[
Z_0 = -j \frac{2\pi}{\lambda_0} \sqrt{k_t^2 - k_0^2},
\]

where \( k_0 = 2\pi/\lambda_0 = 2\pi f/\sqrt{\varepsilon_0\mu_0} \) is the free-space wavenumber and \( k_0 \) is the transverse wavenumber for the direction parallel to the interface (\( t \) stands for transverse and \( n \) for normal direction with respect to the surface of the material sample). The symbols \( \eta_0, \lambda_0, \varepsilon_0, \mu_0 \) correspond to the free-space impedance, wavelength, permittivity and permeability, respectively (\( e^{i\omega t} \) time dependence is suppressed, and \( f \) is the operational frequency). The TM wave impedance of the uniaxial medium is given by:

\[
Z = -j \frac{2\pi}{\varepsilon_n\mu_n} \sqrt{\frac{\varepsilon_n}{\varepsilon_0} k_t^2 - \varepsilon_n\mu_n k_0^2},
\]

where \( (\varepsilon_n, \mu_n) \) are the transverse and normal permittivities, and \( \mu_n \) its transverse permeability. It has been shown [18] that the constituent parameters of the uniaxial medium which can achieve conjugate matching with free space \( Z(k_t) = Z_0(k_t) \) for every \textit{single} mode \( k_t \), should satisfy the Perfectly Matched Layer (PML) rule [19] but with negative real parts of the material parameters, namely:

\[
\varepsilon_n = \mu_n = 1, \quad \varepsilon_n = a - jb, \quad a < 0.
\]  

That is why we call such an effective material Conjugate Matched Layer (CML). The ordinary PML just behaves like a perfect “black body” [16, 19] absorbing solely the propagating modes; on the contrary, CML additionally fully exploits \textit{all} the evanescent waves. Note that the parameter \( b > 0 \) represents losses along the transverse direction, which means that a medium defined by (1) is active along the normal direction (\( \text{Im}[\varepsilon_n] > 0 \)).

2 Near-field energy transfer

If we particularize our research to the test-bed configuration of Figure 1a, namely, a grounded slab excited by a tilted (by the angle \( \theta \)) electric-dipole line source (expression for the incident field of such source can be found e.g. in [20]) at distance \( g \) from the interface, we can find in analytical form the electromagnetic power \( P \) absorbed in the slab. To better understand the variation of \( P \), we consider a slightly perturbed version of the material (1) with \( \varepsilon_n = 1/\varepsilon_n - j\delta = 1/\mu_n - j\delta \) (for \( \delta = 0 \), we have the perfect CML of (1)). The absorbed power \( P \) can be written as a sum of two terms: one expressing the energy transferred via the propagating modes \( P_{\text{prop}} \) (which is not dependent on \( a, b, \delta \)) and another corresponding to the evanescent modes \( P_{\text{evan}} \). If we assume a small perturbation parameter \( \delta \rightarrow 0 \), \( P_{\text{evan}} \) takes the following form [18]:

\[
P_{\text{evan}} \approx P_{\text{prop}} \frac{8|a|}{k_0^4 \pi} \int_{k_0}^{\infty} \frac{k_t^2 (k_t^2 - k_0^2 \sin^2 \theta)}{(k_t^2 - k_0^2)^2} e^{-2\pi \sqrt{k_t^2 - k_0^2}} \times \frac{1 + \text{sgn}(a)}{[1 + \text{sgn}(a)^2 + \delta^2 \left( \frac{k_0^2 \epsilon_0}{2(k_t^2 - k_0^2)} \right)^2]^{1/2}} \, dk_t.
\]

It is remarkable that for the DNG case \( (a < 0 \) the aforementioned quantity behaves like \( P_{\text{evan}} - P_{\text{prop}}/\delta \), which means that it takes very high values when the uniaxial slab exhibits electromagnetic behavior close to that of a perfect CML (1). Furthermore, it should be noted that the sign of the absorbed power \( P_{\text{evan}} \) (and accordingly the overall power \( P = P_{\text{prop}} + P_{\text{evan}} \)) is negative when \( \delta < 0 \); it means that the slab acts as a secondary source and pumps (extremely quickly) energy to the system. In other words, our load (the grounded uniaxial slab) can either absorb or emit energy infinitely fast, if it obeys the rule (1) and simultaneously is slightly lossy \( (\delta > 0) \) or slightly active \( (\delta < 0) \), respectively. It should be stressed that the aforementioned separation between activity and passivity is valid for the chosen dipole excitation; for a more general characterization of the medium, we should check the signs of the imaginary part of the supported propagation constant and the real part of the wave impedance. Alternatively, one can check the \textit{negative definiteness} of the dyadic \( ([\epsilon_n] - [\epsilon_n]^*) \), where \([\epsilon_n]\) is the complex permittivity matrix as a criterion for the structure passivity [21]. Specializing to our example case, the perturbed CML medium is passive for the chosen dipole excitation when \( \delta > 0 \) but as far as any possible excitation is concerned, we should take values obeying the inequality \( \delta > b(a^2 + b^2) \).

Extremely efficient absorption is demonstrated by Figure 2, where the ratio \( P/P_{\text{prop}} \) is represented as a function of \( a \) for various loss parameters \( b > 0 \). We consider two scenarios: one as in Figure 2a, where the rule (1) is followed (not exactly but with a small \( \delta > 0 \)) and one as in Figure 2b, where the effective parameters of the medium are all passive, namely \( \varepsilon_n = \mu_n = a - jb \) but \( \varepsilon_n = \Re[1/\varepsilon_n] = a/(a^2 + b^2) \). In other words, the first case corresponds to a medium with an active normal component but overall the structure absorbs energy from the source \((0 < \delta \rightarrow 0)\) and in the second case we consider a non-active normal component with \( \delta = b(a^2 + b^2) \). The most dominant characteristic of the two graphs is the switch of \( P \) from very low values for \( a > 0 \) (conventional PML “black body” absorption \( P_{\text{prop}} \), double positive DPS medium) to extremely high values in the CML regime \((a < 0)\). It should be stressed that the nonactive case exhibits
high absorbing efficiency \((P \gg P_{\text{prop}})\), which is, however, smaller than in the CML structure. Furthermore, an interesting feature is the effect of \(\delta\) in Figure 2a which makes \(P\) to increase substantially for small \(|a| < 0\); on the contrary, in Figure 2b, where all the material parameters are passive, the power \(P\) is a growing function of \(|a| < 0\).

### 3 Far-field energy transfer

#### 3.1 Single particle

The structure exhibits extremely high efficiency as an emitter, namely has \(P_{\text{evan}} = P_{\text{prop}}\) when choosing a small \(|\delta| < 0\); however, this does not mean that this huge power travels far away. Due to the nature of evanescent fields which are responsible for the development of \(P_{\text{evan}}\), the resonant fields decay exponentially with the distance from the planar interface. This would not be the case if the slab had a finite size, since its evanescent modes would always have a non-zero propagating factor; however, in this work we confine our research to the simple, analytically solvable configurations of Figure 1. In order to remedy this weakness (of the infinite layer) and enable sending a significant portion of that huge \(P\) which is huge regardless of the choice of the sign of \(\delta\) to the far-field region, we add a small cylindrical particle in the near field of the structure (close to \(x = g\) interface). Furthermore, to understand better the effect of this particle, we consider a simple plane wave excitation instead of the tilted dipole of Figure 1a, namely, an incident \(z\)-polarized magnetic field given by:

\[
H_{\text{inc}}(x, y) = \exp[-x^2/k_t^2 - jk_t y].
\]

If the single particle is located at the origin of the coordinate system and is made from a perfect magnetic conductor (a lossless scatterer), the magnetic current \(M\) measured in Volt that would be induced along it, normalized by the corresponding current \(M_0\) in the absence of the CML structure, is written as:

\[
\frac{M}{M_0} = 1 + e^{2\kappa_0(k_t)} \frac{\kappa_0(k_t) - \kappa(\kappa_t)}{\kappa(\kappa_t) + \kappa_t} \int_{0}^{\infty} \frac{e^{i\kappa(t)}(t)}{\sqrt{t}} d \xi.
\]

where \(\kappa_0(k_t) = \sqrt{k_t^2 - k_0^2}\), \(\kappa(k_t) = \sqrt{\frac{k_t^2}{a_x} - \frac{k_0^2}{a_x'}}\), and \(r\) is the radius of the particle. The notation \(H^{(2)}_0\) is used for the Hankel function of zeroth order and second type.
Expression (3) is obtained by assuming that the thickness of the slab is infinite: $k_0L \to +\infty$ (the layer behaves like a half space). The choice of the lossless material (magnetic conductor in this example) of the pin does not play a crucial role; it is made on the basis that renders the formulated boundary value problem simpler. Naturally, the type and the size of the particle of course affect the field distributions, but not decisively. Since the background field is very strong in the region, the induced currents along the cylinder will be rather strong; therefore, it will serve the goal of enhancing the radiation regardless of the kind of object we use, as long as the object is electrically small.

In Figure 3 we show the variation of the magnitude of $M/M_0$ in contour plot with respect to the relative transverse wavenumber $k_t/k_0$ and the real part of the transverse permittivity/permeability $\alpha$ for: (a) $\delta = 0.005 > 0$ and (b) $\delta = -0.005 < 0$. Plot parameters: $b = 0.2$, $k_0r = 0.01$, $k_0g = 0.1$.

In Figure 4 we consider a pin with the electrical radius $k_0r = 0.01$. Again we see how much suppressed gets the current $M$ when $a < 0$ and propagating waves ($k_t < k_0$) are used as an excitation and how mild is the variation when $a > 0$ which confirms that a DPS slab does not enhance substantially the induced current (and accordingly the radiated power) on the perfectly magnetically conducting cylinder. It is noteworthy that $M$ is increased when the reported unlimited
power concentration is developed (DNG and evanescent modes). The same conclusions are drawn from Figure 4b where a thicker dipole ($k_d = 0.05$) is considered. One can observe that $|M/M_0|$ is smaller across the bottom right region of the map compared to Figure 4a; however, that does not necessarily correspond to a lower radiated power since the pin is (five times) more sizeable.

### 3.2 Multiple random particles

By observing the Figures 3 and 4, we remark that the ratio $|M/M_0|$ of the induced currents is not very high, when the structure is excited by a propagating plane wave. The same conclusion is true if we consider as excitation, instead of a plane wave, the dipole of Figure 1a. Reciprocally, if we would use this single pin or a single dipole as a radiating antenna, the presence of a resonant CML close to the small source would not increase radiation into the far zone, since only the evanescent modes will be enhanced. That is why we propose the configuration depicted in Figure 1b where multiple pins (acting as “radiation vessels”) are located in the vicinity of the air-medium interface. That large number of randomly distributed particles, covering a length comparable or larger than the wavelength and positioned in the near field of the structure are able to couple the evanescent fields with the propagating free-space modes and radiate far away from the interface. The considered boundary value problem can be treated with an integral equation formulation [22–24] which fully describes the wave interactions between the source, the CML slab and the cylindrical particles. In this way, one can evaluate the radiated power in the presence ($P_{rad}$) and the absence ($P'_{rad}$) of the particles and accordingly compute the ratio $P_{rad}/P'_{rad}$ which shows how significant is the radiation enhancement.

The dramatic radiative effect of the particles located in the near field of the CML body is shown in Figure 5, where the parameter $P_{rad}/P'_{rad}$ is represented as a function of $a$ for various loss levels $b > 0$. We consider two cases similar to those of Figure 2 in the presence of numerically electrically small cylinders in the vicinity of the interface. Again the proposed solution works only for DNG media ($a < 0$) where extremely high values of the radiation enhancement ratio are observed for the CML case (Figure 5a). It is noteworthy, that for $a > 0$, the radiated far-field is even lower than in the vessel-free structure. Such a result can be explained through partial screening of the propagating modes of the vessel-free structure. In Figure 5b of the nonactive scenario, we notice the same behavior of $P_{rad}/P'_{rad} < 1$ for small negative $a$. It can be attributed to out-of-phase far-field responses of the slab and the particles which are of similar magnitudes due to the weak background field as indicated by Figure 2b. One can observe significant similarities between Figures 2 and 5, which is natural since both effects have the same physical reason: resonant excitation of surface modes. In particular, we have evanescent modes in both regions which are vanishing away from the interface and their nature is plasmonic. Clearly, we have two adjacent half spaces filled with materials whose permittivity has opposite signs (their real parts); therefore, collective electronic excitation is taking place and surface plasmons are excited [25, 26]. In contrast with other works [27–29], we do not care much about the spatial distribution of electromagnetic field owed (or not) to surface modes; we use the developed surface waves along the boundary as “containers” of concentrated energy, which is either absorbed by the CML slab (without particles) or emitted in free-space (with particles).

### 4 Conclusion

To conclude, we revisited the recently introduced concept of CML medium (in both its exact and non-active versions), which leads to huge near field enhancement along its interface, and examined the effect of a single particle placed in its vicinity. The particle acts as a radiation vessel and couples the evanescent modes developed on the CML planar surface with cylindrical radiated waves. We report substantial far-field radiation enhancement when multiple randomly placed particles are positioned close to the CML-vacuum interface. The most important limitation of the proposed idea, since it has not been actually implemented, is imposed by the
difficulties in fabrication of a CML medium. Towards this direction, we are planning the use of fishnet metamaterials (metallic surfaces with holes) or, alternatively, the employment of binary metasurfaces (gratings with two alternating media). In case of success in realizations, it will pave the way for the fabrication of revolutionary, ultra-efficient electronic and photonic designs.


References