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Even-odd mode of a double-Lorentz metamaterial and its application to a tri-band branch-line coupler

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Abstract – The theoretical approach of a double-Lorentz (DL) transmission line (TL) metamaterial using even-odd mode analysis is presented for the application to a tri-band Branch-Line Coupler (BLC). This BLC is based on double-Lorentz (DL) transmission line (TL) metamaterial to achieve the tri-band property. The tri-band operation is achieved by the flexibility in the phase response characteristic of such transmission line. Since metamaterials are in symmetric form, this analysis utilizes superposition and circuit symmetry to solve for the structure's scattering parameters. A design example of a triple band quarter wavelength DL TL suitable for GSM-UMTS applications is designed and evaluated by simulation using even-odd mode analysis to validate the proposed methodology at circuit level. Then, this simulated DL TL is used in the design of a tri-band BLC which is also being analyzed using even-odd mode analysis. This coupler exhibits transmission of 3 ± 0.5 dB, return losses and isolations larger than 14 dB, and a phase difference of ±90 ±3.5°.

Key words: Coupler, Double-Lorentz transmission line, Even mode, Metamaterial, Odd mode, Tri-band component.

1 Introduction

Metamaterials (MTM) are artificial periodic structures with unusual electromagnetic properties fabricated with a negative effective dielectric permittivity and magnetic permeability. This corresponds to a new class named Left-Handed (LH) MTM which have gained significant interest in many guided waves and radiated applications. LH materials are so named because of the LH triad formed by the electric field, magnetic field, and wave vector leading to an antiparallel phase and group velocities [1]. Going through the transmission line (TL) approach, a LH TL is made up of periodic series capacitances and shunt inductances which is the dual of the conventional TL known as Right-handed (RH) TL. But a purely left-handed (LH) TL doesn't exist due to the natural parasitic induced current and voltage which are modeled by a series inductance and a shunt capacitance. This was the motivation for introducing the term CRLH (Composite Right-Left Handed) TL. The dual concept of such CRLH was introduced in [2]. However, the dual CRLH structure is an idealization that cannot be exactly recognized. A real dual CRLH MTM is in fact a double-Lorentz (DL) medium and this material has an intrinsic tri-band property that can be used to design various tri-band microwave components [3]. Both effective material parameters μ_r and ε_r of the corresponding line show Lorentz-type dispersion.

Many microwave components are based on quarter wavelength transmission lines as Branch-Line Couplers (BLC) [4]. But conventional quarter wavelength TLs known as RH TLs can operate only at their desired frequency and odd harmonics. Since wireless communication systems as GSM-UMTS systems have operational non-harmonic frequencies, the conventional BLC can't be an actual solution for them. Metamaterial (MTM) with its unusual properties helped to overcome many problems in the microwave world; one of which is increasing the number of operating frequencies. Tri-band components are helpful to reduce the size and the number of devices used in recent multi-band telecommunication systems [5].

The natural BLC is modified by replacing the conventional transmission lines TLs known as right-handed transmission lines RH TLs with Double-Lorentz DL TLs to have a new one with three arbitrary operating frequencies. The advantage of using DL TLs over RH TLs is shown in the flexibility in the phase response diagram for which we can intercept a desired pair of phases at any arbitrary triple frequencies (f_1, f_2, f_3) for tri-band operation so that f_2 and f_3 are not

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necessary to be multiples of f_1. Tri-band components are helpful to reduce the size and the number of devices used in recent multi-band telecommunication systems.

Two, three, or four port networks symmetric with respect to one or two planes are extremely implemented in RF and microwave devices. In [6], a full design of a DL TL was presented with useful design equations and an implementation of a tri-band branch-line coupler was done using such type of MTM in [7]. Even-odd mode analysis is a classic topic for solving the scattering parameters of a symmetric circuit. A full analysis of symmetrical two port network and four port network is done in [8] and [9] respectively. The implementation of DL TL MTM using circuit models has been well investigated in the past few years but analyzed without taking the symmetrical advantage. However, calculations will be well simplified if a symmetric structure is divided into sub-circuits. Since DL TLs MTM can be implemented using symmetrical model, one can analyze only half the circuit. In [10], an even-odd mode excitation is done for a bi-symmetrical dual-band BLC but not based on MTM. The main objective of this paper is to verify the use of even-odd mode analysis of metamaterial for a two port symmetric balanced structure of a DL TL to be extended in the use of a tri-band BLC which is a symmetrical four port network.

2 Double-Lorentz transmission line metamaterial

2.1 Double-Lorentz transmission line approach

The unit cell of the artificial DL TL consists of lumped elements L_R and C_L that are parallel in the series path and then of L_L and C_R that are series in the shunt path, a parasitic series inductance L_P and a shunt capacitance C_P as shown in Figure 1. A DL TL is designed by cascading periodically this unit cell with a condition that this cell is much smaller than the guided wavelength (λ_g) in the frequency range of operation. Mainly, it is examined in the homogeneous limit where $(\Delta/\lambda_g) \to 0$.

As shown in Figure 1, the unit cell series impedance Z_{se} and shunt admittance Y_{sh} are given by (1) and (2):

$$Z_{se}(\omega) = j\omega L_P \frac{\omega^2 - \omega_0^{se\,2}}{\omega^2 - \omega_\infty^{se\,2}}, \tag{1}$$

where $\omega_\infty^{se} = \frac{1}{\sqrt{L_R C_L}}$ and $\omega_0^{se} = \sqrt{\frac{L_R + L_P}{L_R L_P C_L}}$.

$$Y_{sh}(\omega) = j\omega C_P \frac{\omega^2 - \omega_0^{sh\,2}}{\omega^2 - \omega_\infty^{sh\,2}}, \tag{2}$$

where $\omega_\infty^{sh} = \frac{1}{\sqrt{L_L C_R}}$ and $\omega_0^{sh} = \sqrt{\frac{C_R + C_P}{L_L C_R C_P}}$.

The constitutive parameters μ_{eff} and ε_{eff} are plotted for a specific set of LC parameters in Figure 2.

The DL structure can be balanced so that no gap exists in the transition from LH medium to RH medium. There are two conditions to reach such case:

$$\omega_\infty^{se} = \omega_\infty^{sh} = \omega_\infty \quad \text{and} \quad \omega_0^{se} = \omega_0^{sh} = \omega_0. \tag{3}$$

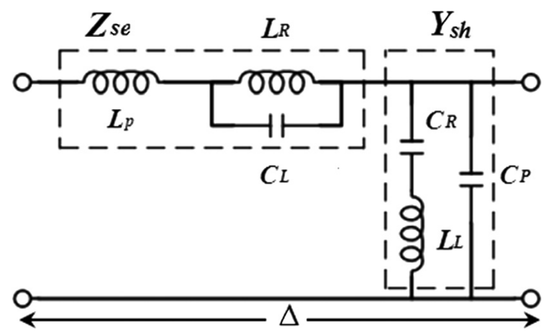

Figure 1. Unit-cell of artificial double-Lorentz (DL) transmission line (TL).

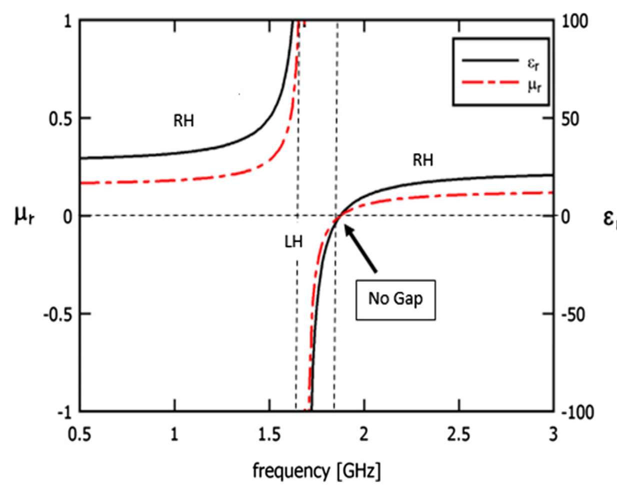

Figure 2. DL TL metamaterial constitutive parameters for a specific set of LC parameters.

Under the balanced condition, the dispersion relation and the characteristic impedance are given by (4) and (5):

$$\beta(\omega)\Delta = \frac{\omega}{\omega_p} \frac{\omega^2 - \omega_0^2}{\omega^2 - \omega_\infty^2}, \tag{4}$$

where $\omega_p = \frac{1}{\sqrt{L_p C_p}}$.

$$Z_0 = \sqrt{\frac{L_R}{C_R}} = \sqrt{\frac{L_L}{C_L}} = \sqrt{\frac{L_P}{C_P}}. \tag{5}$$

2.2 Tri-band design procedure

The TL has six variables L_P, C_P, L_R, C_R, L_L, and C_L that should be calculated first. If we assume that the operating frequencies are chosen as f_1, f_2, and f_3, the phase shift of quarter wavelength DL TL at each frequency is given by (6)–(8):

$$\varphi(f_1) = -\pi/2, \tag{6}$$

$$\varphi(f_2) = +\pi/2, \tag{7}$$

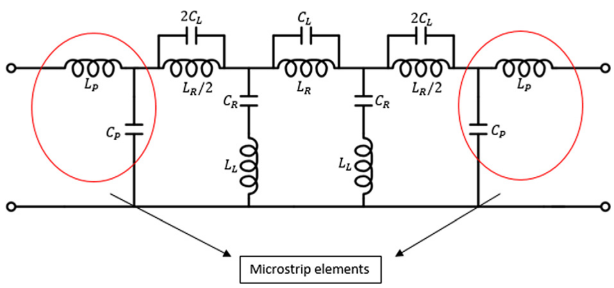

Figure 3. Schematic of a symmetric DL TL with two unit cells.

$$\varphi(f_3) = -\pi/2. \tag{8}$$

The phase shift is related to β by $\varphi_i = -\beta_i N\Delta$ where N is the number of unit cells and $i = (1, 2, 3)$. So, the dispersion relation can be written in the form (9):

$$\frac{-\varphi_i}{N} = \frac{\omega_i}{\omega_p}\frac{\omega_i^2 - \omega_0^2}{\omega_i^2 - \omega_\infty^2} \quad \text{where} \quad i = (1, 2, 3). \tag{9}$$

2.3 Implementation

We noted that a DL TL is obtained by cascading the unit cell shown in Figure 1. However, to have equal input and output impedances, a balanced symmetric structure is recommended instead. A schematic of a symmetric DL TL with two unit cells is shown in Figure 3. Surface Mount Technology is used for the LH and RH components while the sub-section that corresponds to the parasitic elements is implemented using microstrip lines.

The procedure of implementation is summarized as follows:

1. Choose f_1, f_2, and f_3.
2. Solve the system of equations obtained in (9) for the unknown values of ω_0, ω_∞, and ω_p.
3. With the help of ω_0, ω_∞, ω_p, and Z_0, Calculate the values of L_P, C_P, L_L, C_L, L_R, and C_R which are derived to be:

$$L_P = \frac{Z_0}{\omega_p} \quad \text{and} \quad C_P = \frac{1}{\omega_p Z_0}, \tag{10}$$

$$L_R = \frac{Z_0(\omega_0^2 - \omega_\infty^2)}{\omega_p\omega_\infty^2} \quad \text{and} \quad C_R = \frac{(\omega_0^2 - \omega_\infty^2)}{Z_0\omega_p\omega_\infty^2}, \tag{11}$$

$$L_L = \frac{Z_0\omega_p}{(\omega_0^2 - \omega_\infty^2)} \quad \text{and} \quad C_L = \frac{\omega_p}{Z_0(\omega_0^2 - \omega_\infty^2)}. \tag{12}$$

4. Be sure that the operating frequencies are not found in the stop-band in the dispersion diagram between right-handed media at lower frequencies and left-handed media at higher ones. Otherwise, increase the number of unit cells chosen.
5. Use the values of L_P and C_P to find the lengths and widths of the microstrip lines using standard microstrip formulas.

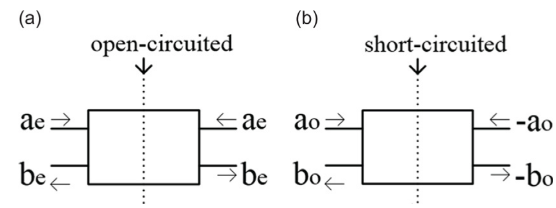

Figure 4. Two port network (a) even-mode excitation (b) odd-mode excitation.

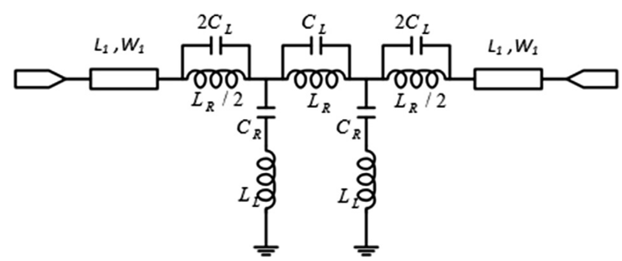

Figure 5. Schematic of a 50 Ω DL TL for: L_1 = 16.25 mm, C_R = 0.55 pF, C_L = 7.24 pF, L_R = 1.205 nH, L_L = 18.24 nH.

3 Even-odd mode analysis of a DL TL

3.1 Symmetrical two-port network

A symmetrical network can be defined by a one having a plane of symmetry. Calculations will be well simplified when a two port network is divided into two structures mirroring each other [11]. This is a main requirement in analyzing complex symmetric structures. When an even excitation is applied to the network, the two applied signals at ports 1 and 2 are in phase. This creates a virtual open circuit symmetrical interface ("magnetic wall"). Similarly, under an odd excitation where the two applied signals are out of phase, the symmetrical interface is a virtual short circuit ("electric wall") as shown in Figure 4.

3.2 Scattering parameters

The network analysis will be simplified by analyzing each one port separately and then determining the two-port network parameters from the even and odd mode network parameters. The two port S-parameters are established where the subscripts "e" and "o" refer to the even mode and odd mode respectively [12]:

$$S_{11} = \frac{1}{2}(S_{11e} + S_{11o}), \tag{13}$$

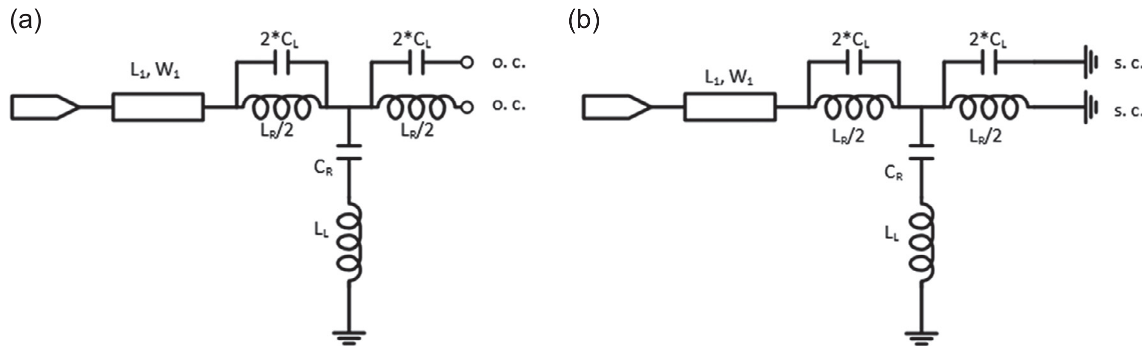

Figure 6. (a) Even mode excitation, (b) odd mode excitation.

$$S_{21} = \frac{1}{2}(S_{11e} + S_{11o}), \quad (14)$$

$$S_{12} = S_{21}\,(\text{by symmetry}), \quad (15)$$

$$S_{22} = S_{11}\,(\text{by symmetry}). \quad (16)$$

3.3 Even-odd mode of a DL TL

A schematic of a symmetric 50 Ω DL TL using the procedure above is shown in Figure 5.

For even mode excitation, we can bisect the network with open circuits at the symmetrical interface as shown in Figure 6a. For odd mode excitation, we can bisect the network with short circuits at the symmetrical interface as shown in Figure 6b.

3.4 Simulation results

After bisecting the DL TL into two symmetric halves and applying the even-odd mode on the obtained two networks, simulation is done to find the S_{11} parameter for each one alone using [13]. However, the reflection coefficient S_{11} and the transmission coefficient S_{21} for the full DL TL can be directly obtained from (13) to (14) respectively and plot in Figure 7. The operating frequencies are 900 MHz, 1800 MHz, and 2100 MHz where the phase response is −90°, +90°, −90° respectively.

4 Even-odd mode analysis of a BLC

4.1 Tri-band BLC

Following the previous procedure in Section 2.3, a BLC is implemented using 50 Ω and 35 Ω DL TLs using the schematic shown in Figure 5. The microstrip substrate used is FR4 with permittivity 4.4, thickness 0.8 mm, and copper thickness 18 μm. The operating frequencies are chosen to be 0.9 GHz, 1.8 GHz, and 2.1 GHz. The frequency dependence of the element components causes variations in the characteristic impedance of the DL TL, which results in an amplitude imbalance between the two output ports. To compensate this effect, a tuning stub is added to the 50 Ω DL TLs preserving the symmetric structure also. The length of the stub is tuned and found to be 2 mm. For more details, see [7].

4.2 Even-odd mode of a tri-band BLC

For a tri-band BLC, the structure will become more complex. To simplify calculation, let us consider the full symmetrical four port network with *XX* and *YY* symmetry axes. This is the case of a bisymmetrical structure where we can decompose the network into four single port sub-circuits (even-even, even-odd, odd-even, and odd-odd) by the double application of the even-odd mode decomposition [14] as shown in Figure 8. The subscript 35 and 50 are used for the 35 Ω and 50 Ω TLs respectively.

The four port *S*-parameters are established as function of the single port networks parameters where the subscripts e and o refer to the even mode and odd mode respectively:

$$S_{11} = \frac{1}{4}(S_{11ee} + S_{11eo} + S_{11oe} + S_{11oo}), \quad (17)$$

$$S_{21} = \frac{1}{4}(S_{11ee} - S_{11eo} + S_{11oe} - S_{11oo}), \quad (18)$$

$$S_{31} = \frac{1}{4}(S_{11ee} - S_{11eo} - S_{11oe} + S_{11oo}), \quad (19)$$

$$S_{31} = \frac{1}{4}(S_{11ee} + S_{11eo} - S_{11oe} - S_{11oo}). \quad (20)$$

4.3 Simulation results

After bisecting the BLC into four symmetric sections and applying equations (1) to (4), the simulated *S*-parameters are shown in Figure 9. The operating frequencies are 900 MHz, 1800 MHz, and 2100 MHz where the phase difference

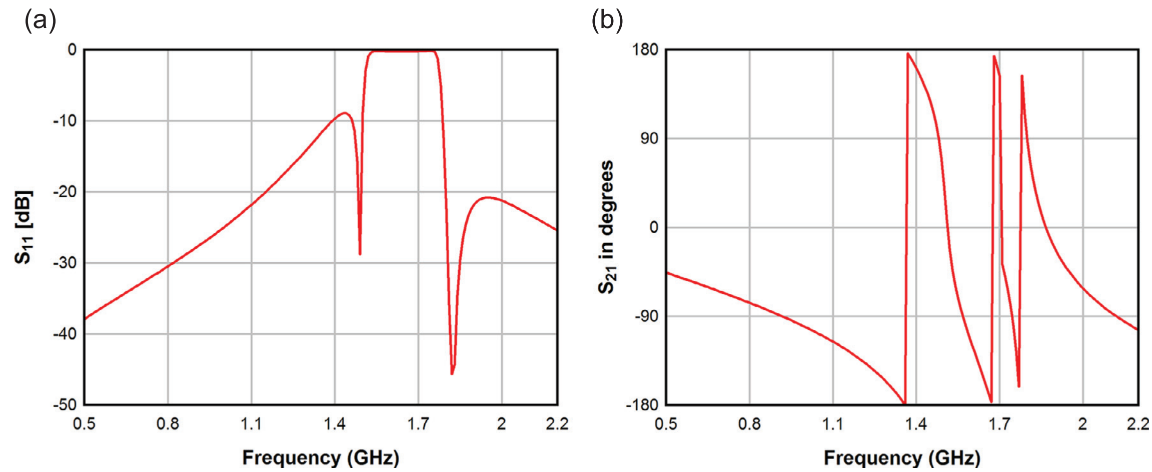

Figure 7. (a) Reflection coefficient of the DL TL using S_{11}-even and S_{11}-odd, (b) phase response S_{21} of the DL TL.

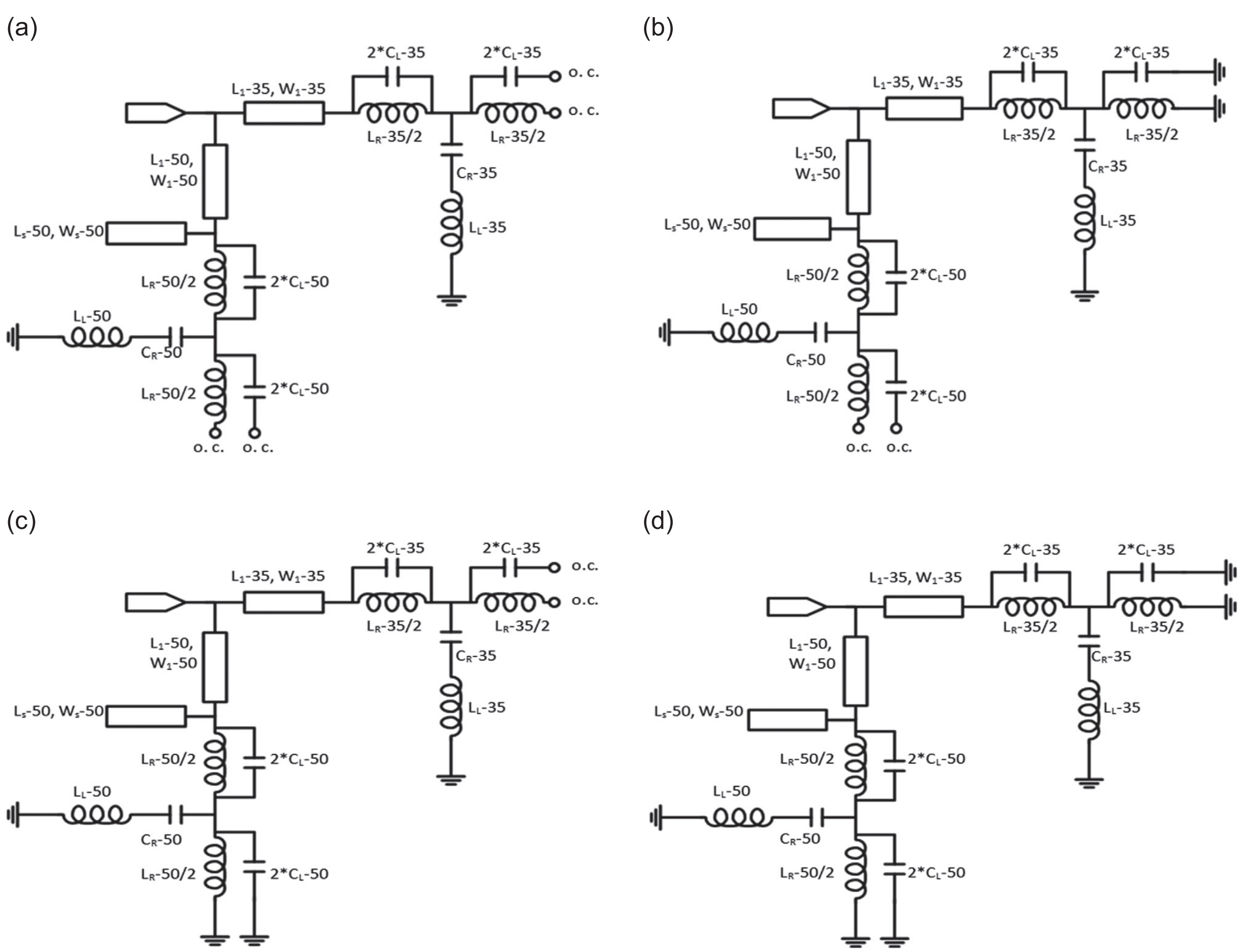

Figure 8. Reduced subcircuits (a) even-even (b) even-odd (c) odd-even (d) odd-odd for L_R-35 = 0.87 nH, C_R-35 = 0.8 pF, L_L-35 = 12 nH, C_L-35 = 10.2 pF, L_1-35 = 16.3 mm, W_1-35 = 2.63 mm, L_R-50 = 1.3 nH, C_R-50 = 0.5 pF, L_L-50 = 18.1 nH, C_L-50 = 7.2 pF, W_1-50 = 1.51 mm, L_1-50 = 17 mm, L_s = 2 mm.

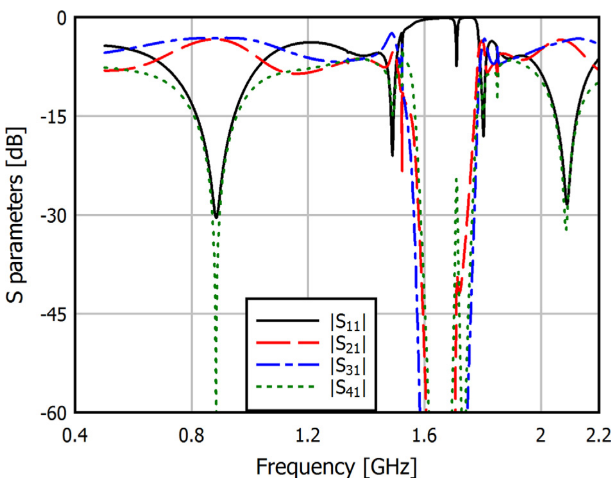

Figure 9. Simulated S-parameters of the tri-band BLC using the single port sub-circuit parameter.

between the two output ports is ±90° ±3.5°. Figure 9 shows that the tri-band is well achieved where return losses as well as isolations are larger than 14 dB at each operating frequency; however, S_{21} and S_{31} are of −3 dB ± 0.5 dB.

5 Conclusion

In this paper, an even-odd mode analysis of a DL TL metamaterial is presented. This DL TL has a tri-band property to be used in the design of tri-band microwave devices. So, this analysis was done to simplify calculations for complex circuits especially to those of periodic structure with much number of units as well as for complex structures used in EM simulators. This analysis has been also illustrated by a 50 Ω, $\lambda/4$ DL TL and a general description of any two port network is given first. Then, we extended our study to the application of a tri-band BLC using also even-odd mode analysis with bisymmetrical symmetry.

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