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Nihility in non-reciprocal bianisotropic media

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Abstract – Here we consider electromagnetic response of non-reciprocal bianisotropic materials in some extreme regimes. The magneto-electric coupling is modeled by symmetric and antisymmetric uniaxial dyadics, which correspond to the so called artificial Tellegen media and moving media, respectively. Extreme electromagnetic properties of uniaxial non-reciprocal bianisotropic materials in the limiting case of *nihility*, when both permittivity and permeability of the media tend to zero, and only the magneto-electric parameters define the material response, are studied. Among other interesting effects, we show that the moving nihility materials provide the extreme asymmetry in the phase shift of transmitted waves propagating along the opposite directions. Furthermore, we reveal a possibility to create an angular filter with extreme sensitivity to the incidence angle, also using moving nihility slabs.

Key words: Bianisotropic media, Wave propagation.

1 Introduction

The metamaterial paradigm offers possibilities to design and realize electromagnetic materials with desired values of constitutive parameters (naturally, with some limitations). It is of theoretical and practical interest to explore the properties of artificial materials with extreme values of material parameters, either very large or very small. Such extreme materials show interesting, often extreme, electromagnetic properties [1]. Recently, materials with very small values of the permittivity and/or permeability attracted attention due to some interesting physical effects and application possibilities, see e.g. [2–4]. General properties of isotropic media with either very small or very large parameter values were discussed in reference [1]. In this paper we study some extreme cases of bianisotropic materials, focusing on the case of non-reciprocal materials.

It is well known that the difference of the plane-wave propagation factors β of the two (circularly polarized) eigenwaves in isotropic chiral media is defined by the chirality parameter κ in the Tellegen formalism as [5] $\beta = k_0\,(N \pm \kappa)$, where $k_0 = \omega\sqrt{\epsilon_0\mu_0}$ is the free-space wavenumber and $N = \sqrt{\epsilon\mu/(\epsilon_0\mu_0)}$ is the refractive index. In the extreme case when $N \rightarrow 0$ but $\kappa \neq 0$, the propagation factors of the two eigenmodes differ by sign, maximizing birefringence of medium. In particular, mixtures of optimal helices [6, 7] with $\kappa \approx 1$ realize effective media with the propagation factor $\beta = -k_0$ for one of the circular polarizations, while the same medium is transparent for the orthogonal circular polarization [8]. This extreme-parameter medium is called *chiral nihility* [9]. The concept of chiral nihility leads to understanding of the chiral route to negative refraction and superlensing with the use of chiral structures [9, 10]. In the context of this work, we can say that the asymmetry in the propagation constants of the two eigenwaves comes to its extreme (the two values differ by sign) in the limiting case of chiral nihility. The wave impedances of the two eigenwaves are always the same as they do not depend on the chirality parameter κ.

More recently, it was understood that there exist optimal parameters (the optimal shapes and sizes of meta-atom inclusions) also for another fundamental class of reciprocal bianisotropic media: omega materials [11]. In contrast to chiral media where the chirality breaks the symmetry of the propagation constants of eigenwaves while the wave impedances are not affected, in omega materials the properties are dual: Magneto-electric coupling breaks the symmetry of the wave impedances of the counter-propagating modes while the propagation constants remain symmetric [12]. It was shown that *omega nihility* media have extreme properties in reflection from a planar slab: the reflection coefficient changes its sign when one or the other face of the slab is illuminated [13].

The goal of this work is to study the fundamental properties of non-reciprocal bianisotropic materials in the case when the permittivity and permeability are negligibly small as compared with the magneto-electric coupling coefficients and understand what kinds of extreme wave properties can be expected for such special materials.

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2 General solution for plane waves propagating in uniaxial non-reciprocal media

In this work we concentrate on the properties of bianisotropic materials in which the magneto-electric coupling is due to non-reciprocal effects. Thus, we assume that the material under study is non-chiral and there is no omega coupling. In this case, the appropriate constitutive relations for non-reciprocal bianisotropic media having uniaxial symmetry (the axis direction is along the unit vector $\mathbf{z}_0$) are described by uniaxial material dyadics as [12]

$$\mathbf{D} = \overline{\overline{\epsilon}} \cdot \mathbf{E} + \sqrt{\epsilon_0\mu_0}\left(\overline{\overline{\chi}}_\mathrm{t} + \chi_\mathrm{n}\mathbf{z}_0\mathbf{z}_0 - \alpha\overline{\overline{J}}_\mathrm{t}\right) \cdot \mathbf{H},$$
$$\mathbf{B} = \overline{\overline{\mu}} \cdot \mathbf{H} + \sqrt{\epsilon_0\mu_0}(\overline{\overline{\chi}}_\mathrm{t} + \chi_\mathrm{n}\mathbf{z}_0\mathbf{z}_0 + \alpha\overline{\overline{J}}_\mathrm{t}) \cdot \mathbf{E}. \quad (1)$$

Here, $\overline{\overline{\epsilon}} = \epsilon_0(\epsilon_\mathrm{t}\overline{\overline{I}}_\mathrm{t} + \epsilon_\mathrm{n}\mathbf{z}_0\mathbf{z}_0)$ and $\overline{\overline{\mu}} = \mu_0(\mu_\mathrm{t}\overline{\overline{I}}_\mathrm{t} + \mu_\mathrm{n}\mathbf{z}_0\mathbf{z}_0)$ are the material permittivity and permeability, $\overline{\overline{I}}_\mathrm{t} = \overline{\overline{I}} - \mathbf{z}_0\mathbf{z}_0$ is the transverse unit dyadic, and $\overline{\overline{J}}_\mathrm{t} = \mathbf{z}_0 \times \overline{\overline{I}}_\mathrm{t}$ is the 90 degree rotator in the $x - y$ plane. Parameters α and $\chi_{(\mathrm{t,\ n})}$ measure the strength of magneto-electric coupling and are called the effective velocity (in artificial moving media) and the Tellegen parameter, respectively. For lossless materials, these coupling parameters are real-valued.

In this paper we will derive field equations, study the propagation factors and the wave impedances of the eigenwaves in the general case of uniaxial non-reciprocal bianistropic media, and then consider the extreme cases of nihility materials. In the following derivations we use the general method for analysing plane waves in uniaxial materials, described in [12, 14]. We look for time-harmonic plane-wave solutions of the Maxwell equations

$$\nabla \times \mathbf{E} = -j\omega\mathbf{B}, \quad \nabla \times \mathbf{H} = j\omega\mathbf{D} \quad (2)$$

in form $e^{-j\mathbf{k}\cdot\mathbf{r}}$, where the wavevector $\mathbf{k} = \beta\mathbf{z}_0 + \mathbf{k}_\mathrm{t}$ (with $\mathbf{z}_0 \cdot \mathbf{k}_\mathrm{t} = 0$). As we are considering structures with the uniaxial symmetry, it is convenient to split the field vectors into the longitudinal and transverse parts with respect to the material axis $\mathbf{z}_0$:

$$\mathbf{E} = E_\mathrm{n}\mathbf{z}_0 + \mathbf{E}_\mathrm{t}, \quad \mathbf{H} = H_\mathrm{n}\mathbf{z}_0 + \mathbf{H}_\mathrm{t} \quad (3)$$

(here the vectors marked by index t are orthogonal to $\mathbf{z}_0$). Substituting these fields into the Maxwell equations and performing the two-dimensional Fourier transform in the transverse plane ($x - y$), after some mathematical manipulations, we arrive at a system of two equations for the transverse components of electric and magnetic fields:

$$\left[\left(\frac{\partial}{\partial z} + jk_0\alpha\right)\overline{\overline{I}}_\mathrm{t} - j\left(k_0\chi_\mathrm{t}\overline{\overline{I}}_\mathrm{t} - \frac{\chi_\mathrm{n}}{k_0(\epsilon_\mathrm{n}\mu_\mathrm{n} - \chi_\mathrm{n}^2)}\mathbf{k}_\mathrm{t}\mathbf{k}_\mathrm{t}\right) \cdot \overline{\overline{J}}_\mathrm{t}\right] \cdot \mathbf{E}_\mathrm{t}$$
$$= \left(j\omega\mu_0\mu_\mathrm{t}\overline{\overline{I}}_\mathrm{t} - \frac{j\eta_0\mu_\mathrm{n}}{k_0(\epsilon_\mathrm{n}\mu_\mathrm{n} - \chi_\mathrm{n}^2)}\mathbf{k}_\mathrm{t}\mathbf{k}_\mathrm{t}\right) \cdot \mathbf{z}_0 \times \mathbf{H}_\mathrm{t},$$
$$\left[\left(\frac{\partial}{\partial z} + jk_0\alpha\right)\overline{\overline{I}}_\mathrm{t} + \overline{\overline{J}}_\mathrm{t} \cdot \left(jk_0\chi_\mathrm{t}\overline{\overline{I}}_\mathrm{t} - \frac{j\chi_\mathrm{n}}{k_0(\epsilon_\mathrm{n}\mu_\mathrm{n} - \chi_\mathrm{n}^2)}\right)\right] \cdot \mathbf{z}_0 \times \mathbf{H}_\mathrm{t}$$
$$= \left[j\omega\epsilon_0\epsilon_\mathrm{t}\overline{\overline{I}}_\mathrm{t} - \frac{j\epsilon_\mathrm{n}}{\eta_0 k_0(\epsilon_\mathrm{n}\mu_\mathrm{n} - \chi_\mathrm{n}^2)}\mathbf{z}_0 \times \mathbf{k}_\mathrm{t}\mathbf{z}_0 \times \mathbf{k}_\mathrm{t}\right] \cdot \mathbf{E}_\mathrm{t}. \quad (4)$$

Here $\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$ is the free-space wave impedance, and $\mathbf{k}_\mathrm{t}$ stands for the two-dimensional Fourier variable (the transverse component of the propagation factor). Eliminating the magnetic field from (4), we come up with a second-order equation for the transverse electric field component $\mathbf{E}_\mathrm{t}$:

$$\left(a_{11}\frac{\mathbf{k}_\mathrm{t}\mathbf{k}_\mathrm{t}}{k_\mathrm{t}^2} + a_{22}\frac{\mathbf{z}_0 \times \mathbf{k}_\mathrm{t}\mathbf{z}_0 \times \mathbf{k}_\mathrm{t}}{k_\mathrm{t}^2} + a_{12}\frac{\mathbf{k}_\mathrm{t}\mathbf{z}_0 \times \mathbf{k}_\mathrm{t}}{k_\mathrm{t}^2} + a_{21}\frac{\mathbf{z}_0 \times \mathbf{k}_\mathrm{t}\mathbf{k}_\mathrm{t}}{k_\mathrm{t}^2}\right) \cdot \mathbf{E}_\mathrm{t} = 0, \quad (5)$$

where the coefficients read

$$a_{11} = \frac{nk_0^2}{\mu_\mathrm{t}nk_0^2 - \mu_\mathrm{n}k_\mathrm{t}^2}\left(\frac{1}{k_0}\frac{\partial}{\partial z} + j\alpha\right)^2 - \frac{\chi_\mathrm{t}^2}{\mu_\mathrm{t}} + \epsilon_\mathrm{t},$$
$$a_{22} = \frac{1}{\mu_\mathrm{t}}\left(\frac{1}{k_0}\frac{\partial}{\partial z} + j\alpha\right)^2 - \frac{nk_0^2}{\mu_\mathrm{t}nk_0^2 - \mu_\mathrm{n}k_\mathrm{t}^2}\left(\frac{\chi_\mathrm{n}k_\mathrm{t}^2}{nk_0^2} - \chi_\mathrm{t}\right)^2 - \frac{\epsilon_\mathrm{n}k_\mathrm{t}^2}{nk_0^2} + \epsilon_\mathrm{t},$$
$$a_{12} = a_{21} = -j\left[\frac{nk_0^2}{\mu_\mathrm{t}nk_0^2 - \mu_\mathrm{n}k_\mathrm{t}^2}\left(\frac{\chi_\mathrm{n}k_\mathrm{t}^2}{nk_0^2} - \chi_\mathrm{t}\right) + \frac{\chi_\mathrm{t}}{\mu_\mathrm{t}}\right]\left(\frac{1}{k_0}\frac{\partial}{\partial z} + j\alpha\right), \quad (6)$$

with $n = \varepsilon_\mathrm{n}\mu_\mathrm{n} - \chi_\mathrm{n}^2$. To derive the wave propagation factors β, we assume a plane-wave solution in the form $\exp(-j\beta z)$ and equate the determinant of (5) to zero. We get a bi-quadratic eigenvalue equation for the propagation factor β:

$$\left(\frac{\beta}{k_0} - \alpha\right)^4 - \left[2\chi_\mathrm{t}\left(\frac{\chi_\mathrm{n}k_\mathrm{t}^2}{nk_0^2} - \chi_\mathrm{t}\right) - \frac{\mu_\mathrm{t}\epsilon_\mathrm{n}k_\mathrm{t}^2}{nk_0^2} + \mu_\mathrm{t}\epsilon_\mathrm{t} + \epsilon_\mathrm{t}\left(\mu_\mathrm{t} - \frac{\mu_\mathrm{n}k_\mathrm{t}^2}{nk_0^2}\right)\right]\left(\frac{\beta}{k_0} - \alpha\right)^2$$
$$+ n_\mathrm{t}\left[-\frac{\chi_\mathrm{n}^2k_\mathrm{t}^4}{n^2k_0^4} - \chi_\mathrm{t}^2 + \frac{2\chi_\mathrm{t}\chi_\mathrm{n}k_\mathrm{t}^2}{nk_0^2} - \frac{k_\mathrm{t}^2}{nk_0^2}(\epsilon_\mathrm{n}\mu_\mathrm{t} + \epsilon_\mathrm{t}\mu_\mathrm{n}) + \epsilon_\mathrm{t}\mu_\mathrm{t} + \frac{\epsilon_\mathrm{n}\mu_\mathrm{n}k_\mathrm{t}^4}{n^2k_0^4}\right] = 0, \quad (7)$$

where $n_\mathrm{t} = \varepsilon_\mathrm{t}\mu_\mathrm{t} - \chi_\mathrm{t}^2$. Solutions of this equation for the wave propagation factor β read

$$\beta = k_0\alpha \pm k_0\sqrt{-\chi_\mathrm{t}\left(\chi_\mathrm{t} - \frac{\chi_\mathrm{n}k_\mathrm{t}^2}{nk_0^2}\right) + \epsilon_\mathrm{t}\mu_\mathrm{t} - \frac{k_\mathrm{t}^2}{2nk_0^2}(\epsilon_\mathrm{t}\mu_\mathrm{n} + \epsilon_\mathrm{n}\mu_\mathrm{t}) \pm \frac{1}{2}\sqrt{\Delta}},$$
$$\Delta = \frac{k_\mathrm{t}^4}{n^2k_0^4}\left\{(\epsilon_\mathrm{t}\mu_\mathrm{n} - \epsilon_\mathrm{n}\mu_\mathrm{t})^2 + 4\left[\epsilon_\mathrm{t}\mu_\mathrm{t}\chi_\mathrm{n}^2 + \epsilon_\mathrm{n}\mu_\mathrm{n}\chi_\mathrm{t}^2 - \chi_\mathrm{t}\chi_\mathrm{n}(\epsilon_\mathrm{t}\mu_\mathrm{n} + \epsilon_\mathrm{n}\mu_\mathrm{t})\right]\right\}. \quad (8)$$

To get a complete description of eigenwaves, we also need to find the wave impedances for waves travelling in these materials. Dyadic wave impedances define the relations between the transverse electric and magnetic fields components in plane waves propagating in an unbounded medium. We define the relation between transverse electric and magnetic fields as

$$\mathbf{E}_\mathrm{t} = -\overline{\overline{Z}} \cdot \mathbf{z}_0 \times \mathbf{H}_\mathrm{t}. \quad (9)$$

In this definition, the eigenvalues of $\overline{\overline{Z}}$ which have positive real parts correspond to waves propagating along the z-axis, and eigenvalues having negative real part correspond to eigenwaves propagating in the opposite direction. For the four eigensolutions, the wave impedances can be readily found from (4) by substituting the corresponding propagation factors from (8). After some dyadic algebra we find

$$\overline{\overline{Z}} = \frac{\eta_0}{k_0\epsilon_\mathrm{t}}\left\{(\beta - k_0\alpha)\frac{\mathbf{k}_\mathrm{t}\mathbf{k}_\mathrm{t}}{k_\mathrm{t}^2} + \frac{n\epsilon_\mathrm{t}k_0^2}{n\epsilon_\mathrm{t}k_0^2 - \epsilon_\mathrm{n}k_\mathrm{t}^2}(\beta - k_0\alpha)\frac{\mathbf{k}_\mathrm{t} \times \mathbf{z}_0\mathbf{k}_\mathrm{t} \times \mathbf{z}_0}{k_\mathrm{t}^2}\right.$$
$$\left. - k_0\left[\frac{\epsilon_\mathrm{t}(n\chi_\mathrm{t}k_0^2 - \chi_\mathrm{n}k_\mathrm{t}^2)}{n\epsilon_\mathrm{t}k_0^2 - \epsilon_\mathrm{n}k_\mathrm{t}^2}\frac{\mathbf{z}_0 \times \mathbf{k}_\mathrm{t}\mathbf{k}_\mathrm{t}}{k_\mathrm{t}^2} + \chi_\mathrm{t}\frac{\mathbf{k}_\mathrm{t}\mathbf{k}_\mathrm{t} \times \mathbf{z}_0}{k_\mathrm{t}^2}\right]\right\}, \quad (10)$$

where β is given by (8).

3 Tellegen and moving nihility

In the previous section, we derived the general solutions for the propagation constant and wave impedance in non-reciprocal uniaxial bianisotropic media. Next, we will study what extreme properties, we can expect if the permittivity and permeability of the materials are negligibly small (*bianisotropic nihility*).

3.1 Tellegen nihility

To find the wave propagation factor for the case of Tellegen nihility, we just need to substitute $\alpha = 0$ in (8). Let us consider the Tellegen nihility case where ϵ_t, ϵ_n, μ_t, and μ_n tend to zero. In this case the wave propagation constant reads

$$\beta = \mp jk_0\sqrt{\chi_t\left(\chi_t + \frac{k_t^2}{\chi_n k_0^2}\right)}. \tag{11}$$

As it can be seen from this equation, for lossless media the propagation constants can be purely real, purely imaginary, or zero. For the cases where $\beta \neq 0$, for arbitrary propagation directions all the components of the wave impedance dyadics tend to infinity meaning that these modes do not couple to free-space modes at the sample boundaries. For the case where $\beta = 0$, all the components of the wave impedance dyadics tend to infinity except the component corresponding to $\mathbf{z}_0 \times \mathbf{k}_t\mathbf{k}_t/k_t^2$ which tends to zero.

Another interesting extreme case is the case where only the transverse components of permittivity and permeability tend to zero ϵ_t, $\mu_t \to 0$ but the normal components of the permittivity and permeability dyadics may have any arbitrary non-zero values. The propagation constant for this case reads

$$\beta = \mp jk_0\sqrt{\chi_t\left(\chi_t - \frac{\chi_n k_t^2}{nk_0^2}\right) \mp \frac{k_t^2}{nk_0^2}\chi_t\sqrt{\varepsilon_n\mu_n}}. \tag{12}$$

In this case, for some design parameters, the propagation constants can even take real values. All the components of the wave impedance dyadics tend to infinity except $\mathbf{z}_0 \times \mathbf{k}_t\mathbf{k}_t/k_t^2$ component which gets a purely real value. This interesting wave-propagation phenomenon in Tellegen media happens only in the case of non-axial propagation. For axial propagation, the propagation constants are purely imaginary and all components of the wave impedance dyadics tend to infinity.

3.2 Moving nihility

For the case of an artificial moving medium, by substituting $\chi_t = \chi_n = 0$ in (8), the four values of the propagation constant are given by

$$\beta = k_0\alpha \pm k_0\sqrt{\epsilon_t\mu_t - \frac{k_t^2}{k_0^2}\frac{\mu_t}{\mu_n}} \quad \text{and}$$
$$\beta = k_0\alpha \pm k_0\sqrt{\epsilon_t\mu_t - \frac{k_t^2}{k_0^2}\frac{\epsilon_t}{\epsilon_n}}. \tag{13}$$

Now let us consider the moving nihility case where ϵ_t, ϵ_n, μ_t, and μ_n tend to zero with the same rate. In the case of oblique propagation ($k_t \neq 0$), the wave propagation factors read

$$\beta = k_0\alpha \pm jk_t \tag{14}$$

while the wave impedances are

$$\overline{\overline{Z}} = \infty\frac{\mathbf{k}_t\mathbf{k}_t}{k_t^2} + \infty\frac{\mathbf{k}_t \times \mathbf{z}_0\mathbf{k}_t \times \mathbf{z}_0}{k_t^2} + 0\frac{\mathbf{z}_0 \times \mathbf{k}_t\mathbf{k}_t}{k_t^2} + 0\frac{\mathbf{k}_t\mathbf{k}_t \times \mathbf{z}_0}{k_t^2}, \tag{15}$$

however, for the case of axial propagation ($k_t = 0$), the propagation factors and wave impedances simplify to

$$\beta = k_0\alpha, \qquad \overline{\overline{Z}} = \pm\eta_0\overline{\overline{I}}_t. \tag{16}$$

These interesting results show that such a medium can support both forward and backward waves propagating along the axis. In other words, it behaves as a Veselago medium [15] for waves propagating in one direction along the axis and as vacuum for waves propagating in the opposite direction. Let us consider a material slab of thickness d in free space, filled with a moving nihility material under illumination of a normally incident plane wave. Such a layer is matched for waves hitting any of its sides, however, the transmission coefficient T is different:

$$R = 0, \quad T = e^{\pm jk_0\alpha d}. \tag{17}$$

Similar results were already discovered for metasurfaces with moving magneto-electric coupling [16].

Another interesting example is the case where only transverse components of permittivity and permeability tend to zero. The propagation constant equals

$$\beta = k_0\alpha \tag{18}$$

while the wave impedance becomes

$$\overline{\overline{Z}} = \infty\frac{\mathbf{k}_t\mathbf{k}_t}{k_t^2} + \infty\frac{\mathbf{k}_t \times \mathbf{z}_0\mathbf{k}_t \times \mathbf{z}_0}{k_t^2} + 0\frac{\mathbf{z}_0 \times \mathbf{k}_t\mathbf{k}_t}{k_t^2} + 0\frac{\mathbf{k}_t\mathbf{k}_t \times \mathbf{z}_0}{k_t^2}. \tag{19}$$

For the axial propagation the wave impedance equals to that of free space for both propagation directions:

$$\overline{\overline{Z}} = \pm\eta_0\overline{\overline{I}}_t. \tag{20}$$

It is seen that for axially propagating plane waves in a moving nihility medium we have η_0 wave impedance for waves propagating in both directions, while for waves propagating in any other direction the diagonal components of the impedance dyadics tend to infinity. A slab of such material constitutes an ideal angular filter.

4 Conclusion

Exotic and extreme properties of non-reciprocal nihility materials have been considered. We have studied Tellegen and moving nihility media. It was found that, for specific material parameters, oblique wave propagation is possible in a Tellegen nihility medium while axial propagation is not possible. It was shown that lossless moving nihility materials exhibit extreme asymmetry in the phase propagation constants for oppositely directed waves, so that the phase shift for waves transmitted through a slab in one direction is negative with respect to the phase shift for the oppositely-bound waves. We have also revealed an effect of extreme sensitivity of the reflection coefficient on the incidence angle for moving nihility slabs. It was found that the rate in which ϵ and μ tend to zero is, for some cases, a factor which determines the electromagnetic response of the composite.

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