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Review

Negative refraction, light pressure and attraction, equation $E = mc^2$ and wave-particle dualism

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Abstract – This paper is aimed to illustrate the relation of concepts, such as negative refraction, light pressure and attraction, mass–energy equivalence and wave-particle dualism. At first glance, those physics concepts have nothing in common. However, on closer examination it appears that they are closely linked, although this relationship was clearly shown only relatively recently, when a scientific development introduced the concept of negative refraction.

Key words: Negative Refraction, Light pressure and attraction, Mass–energy equivalence and Wave-particle dualism.

The concepts, listed in the title of this article, at first glance have nothing (or almost nothing?) in common. However, on closer examination it appears that they are closely linked, although this relationship was clearly shown only relatively recently, when a scientific development introduced the concept of negative refraction. This latter phenomenon is realized in the isotropic material, when the phase and group velocity of the wave are directed antiparallel, or, equivalently, are oppositely directed wave vector k and Poynting vector S. In this case well-known Snell law is written in his usual form in terms of incident angle φ, the angle of refraction ϕ and refractive index n:

$$\sin \varphi / \sin \phi = n \tag{1}$$

but the value n is negative [1]. If we assume that the group velocity v_{gr} of the wave which coincides with the direction of the Poynting vector, is always positive, as directed from the emitter to the receiver, phase velocity v_{ph} and wave vector k in the case of a negative refractive index should also be considered negative. Recognition of the possible negative value of k (if positive S) immediately raises the question of the direction of the force exerted by the absorbed or reflected waves on a body placed in a medium with negative refraction. Indeed, in this case the linear momentum of wave, equal to:

$$P = hk \tag{2}$$

is negative, that does not correspond to the usual case of light pressure, but to light attraction [1, 2]. Another problem caused by the fact of negative refraction is associated with the value of mass transferred from the transmitter to the receiver in the event that radiation propagates not in the vacuum, but within a material medium with a non-unity refractive index n, in particular, if n is negative. In this case, carried mass connected with transferred energy not by well-known relation:

$$E = mc^2 \tag{3}$$

but with equation:

$$E = m v_{ph} v_{gr} \tag{4}$$

where v_{ph} is the phase velocity of the wave, and v_{gr} – the group velocity [2]. From this relation it would seem paradoxical conclusion that in case of a negative n when the phase and group velocities are antiparallel, the energy transferred from the emitter to the receiver is not accompanied by mass transfer from the emitter to the receiver, but in the opposite direction – away from the receiver to the source. It would seem that this conclusion contradicts the generally accepted statement that any transfer of energy E must be accompanied by mass transfer in accordance with equation (3).

It is appropriate to recall that the relation (3) relates to the mass transported only with portable energy, not affecting the linear momentum of transferred mass. To clarify this fact we, following [3], consider this process from a different point of view, based primarily on the laws of conservation of energy and momentum.

First let's inert body (called the "radiator") with a rest mass M_0, rest energy $E_0 = M_0c^2$, and linear momentum $P_0 = 0$ emits ("radiates", "throws", "shoots") a particle (body, photon, wave packet) with energy $E_1 \ll E_0$, speed $v_1 \ll c$ and linear

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momentum P_1. Let us not discuss at the beginning the relationships between E_1, P_1 and v_1.

After the emission of a particle, emitter with the mass M_0 begins to move with speed:

$$v_0 = P_1/M_0 \quad (5)$$

in the direction opposite to the direction of linear momentum of emitted particle.

Let us now assume that at the distance L from the emitter is another body, called as receiver, and after some time t the particle emitted by the emitter reaches the receiver and blends (absorbed) with him, changing its mass.

Obviously, time t will be equal:

$$t = L/v_1 \quad (6)$$

During this time the emitter will move at a distance Δx equal to:

$$\Delta x = v_0 t = P_1 L/M_0 v_1 \quad (7)$$

This equation could be represented as:

$$\Delta x M_0 = P_1 L/v_1 \quad (8)$$

Taking in the account stable position of center of mass of the entire system "emitter-emitted particles-receiver", it is obvious that equation (8) should be understood that during the time t emitter with the mass M_0 will move in one direction at a distance Δx, and the emitted particle will move in the opposite direction on distance L and changes mass of the receiver on the value P_1/v_1.

Now, let us refine the specific nature of the emitted particles. First, let us consider the usual material particles, the energy and linear momentum which could are the subject of Lorentz transformations. Then, we write the relation of the particle energy and its linear momentum in the form:

$$P_1 = E_1 v_1/c^2 \quad (9)$$

and substitution of this expression in equation (8) gives:

$$\Delta x M_0 = E_1 L/c^2 \quad (10)$$

From this relation follows at once well-known fact that the change in the rest energy of the body on the value of E_1 leads to a change in its resting mass on the value $M = E_1/c^2$. Now consider the case when the emitter emits a wave (wave packet, the light pulse, the photon) with energy E_1. The linear momentum of the emitted wave is equal:

$$P_1 = E_1/v_{ph} \quad (11)$$

A substitution of equation (11) into (8) gives:

$$\Delta x M_0 = EL/v_{ph} v_1 \quad (12)$$

If we assume that the wave velocity v_1 in this ratio is really the group velocity v_{gr}, then we immediately obtain from equation (4).

From equation (4) follows that in media with negative refraction, radiation transfers the mass not from the transmitter to receiver, but rather from the receiver to the transmitter.

In addition, from the above conclusion follows, that the mass transferred from the transmitter to the receiver is determined not only by transferred energy, but also by transferred linear momentum, which may be associated with energy in different ways, such as is seen from the relation (9) or (11). Thus, the well-known relation (3) is only special case of equation (4).

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