

Bandgap engineering for phononic crystals adjustment and applications

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Abstract. This review focuses on cutting-edge research directions in phononic crystals (PnCs), such as bandgap broadening and bandgap tunability. By elucidating the underlying physical mechanisms, it systematically compares the acoustic performance of different structural configurations. Furthermore, it explores in depth the application potential and specific case studies of such materials in fields including marine engineering. The core advantage of PnCs stems from their artificially engineered sub-wavelength structures, which enable anomalous modulation of constitutive parameters such as effective mass density and elastic modulus. This allows for precise acoustic impedance matching or mismatching, thereby efficiently controlling sound wave propagation paths and energy dissipation. Research indicates that the advancement of PnCs will significantly expand the technical approaches for ship vibration and noise reduction. Notably, with the deep integration of smart materials and digital design, PnCs are gradually evolving from static, passive structural units into dynamic, adaptive, and even intelligent acoustic processing systems. This trend not only provides transformative solutions to the long-standing challenges of low-frequency and ultra-low-frequency noise control but also heralds a new paradigm for acoustic-vibration environment regulation. In the future, major equipment and quiet spaces are expected to achieve active, precise, and programmable management of the acoustic-vibration environment, with a control logic analogous to that of electromagnetic spectrum regulation.

Keywords: Phonic crystals / bad gap property / Vibration and Noise Control

1 Introduction

As a ubiquitous environmental stressor, noise significantly diminishes both the quality of life and work efficiency. The acceleration of industrialization and urbanization had intensified noise pollution, which presented immediate risks to public health and posed substantial challenges to sustainable societal progress. Therefore, a thorough investigation of the etiology, propagation dynamics, and mitigation strategies for noise pollution was imperative to foster improved living conditions and enhance overall quality of life [1–4].

Conventional vibration control strategies, including vibration isolation, dynamic vibration absorption, and damped vibration absorption, exhibited significant limitations. Vibration isolation often required bulky designs and demonstrated poor low-frequency performance.

Dynamic vibration absorbers depended on substantial auxiliary mass, resulting in a narrow effective frequency band. Similarly, damped vibration absorption proved inadequate for mitigating low-frequency vibrations. Collectively, these methods fell short when addressing the challenges posed by complex engineering structures and low-frequency noise. This underscored the critical need for novel artificial micro-structured materials capable of efficiently controlling such wave propagation.

Currently, advancements in materials science and acoustics had catalyzed the emergence of novel artificial materials, notably acoustic metamaterials and phononic crystals (PnCs). The typical structural forms and band gaps of various PnCs are shown in Figure 1. In the 1990s, the concept of PnCs was proposed by analogy to photonic crystals. PnCs [5, 6], as artificially periodic microstructure materials, can exhibit bandgap properties. Acoustic or elastic waves within the bandgap frequency range cannot propagate through PnCs. This characteristic endows PnCs with immense application value in scenarios such as noise reduction, vibration damping, filtering, and waveguiding.

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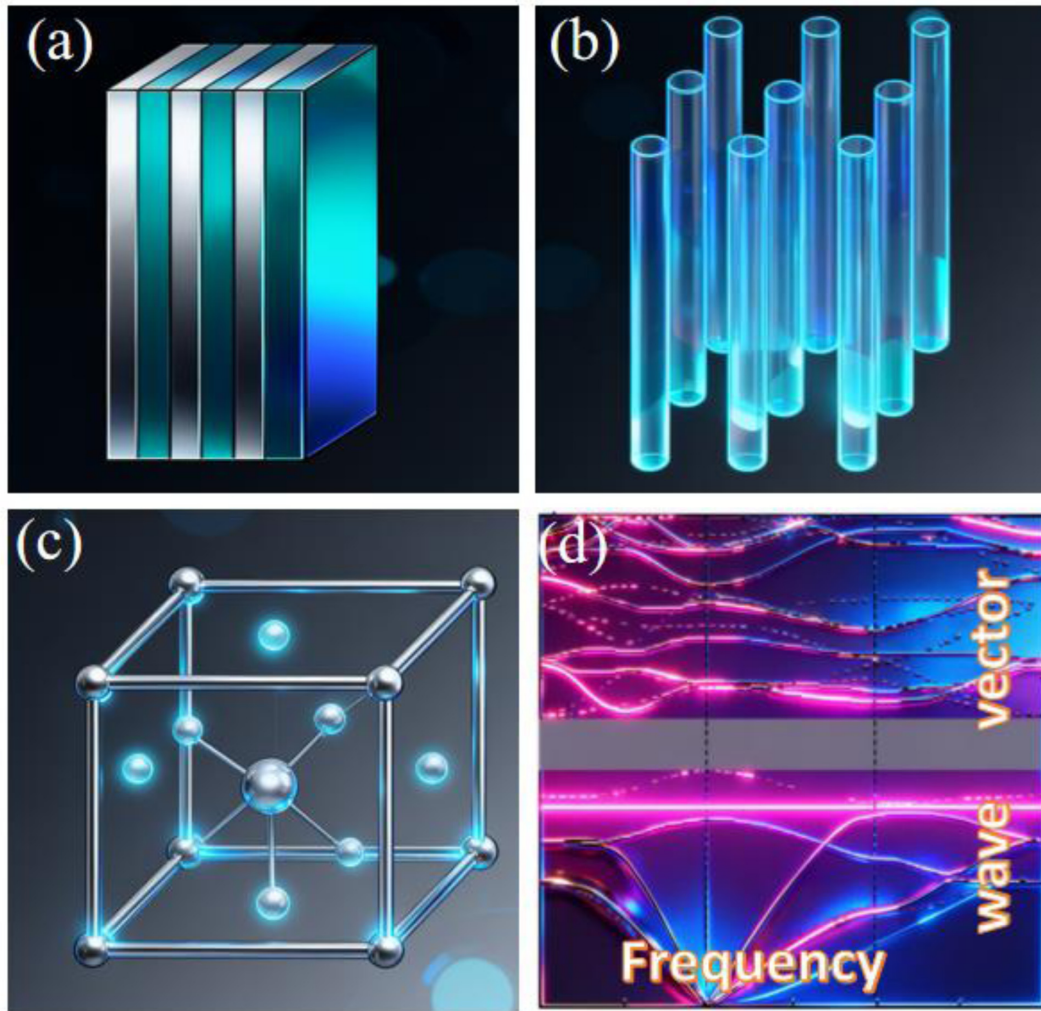


Fig. 1. Phonon crystal and bandgap schematic diagram. (a) One-dimensional phonon crystal. (b) Two-dimensional phonon crystal. (c) Three-dimensional phonon crystal. (d) Bandgap diagram.

Following the precedent set by photonic crystals, where wave propagation is governed by tailored structural design, Kushwaha et al. [7], introduced the foundational concept of PnCs in a groundbreaking 1993 paper. They formally defined PnCs as periodic elastic composites that exhibit absolute phononic bandgaps. The discovery that elastic waves propagating through periodic elastic composite media also demonstrate bandgap effects formed a critical foundation. This implied that the path of elastic waves could be controlled by strategically designing the internal architecture and defects of PnCs, thereby enabling targeted vibration isolation and noise suppression. Early PnCs primarily relied on the Bragg scattering principle [7–10]. When the wavelength of the incident wave approached the lattice constant, significant attenuation of the transmitted wave occurred due to wave interference. Theoretically, in crystals with infinite periodicity, wave propagation within a specific frequency range was entirely prohibited; this range was defined as the band gap or stop band. For actual structures with finite periodicity, the band gap was conventionally identified when the attenuation of the transmitted wave relative to the incident wave

reached a predetermined threshold. The underlying Bragg scattering mechanism implied that controlling low-frequency waves required PnCs with proportionally large lattice constants. This requirement, combined with the necessity for a sufficient number of unit cells to achieve effective attenuation, often resulted in impractically large overall dimensions, thereby hindering practical engineering application. This limitation was addressed in 2000 when Liu pioneered the concept of locally resonant PnCs [11]. In such PnCs, the bandgap was governed by the resonant properties of the constituent units, with lattice constants considerably smaller than the relevant wavelength. This feature enabled the realization of low-frequency bandgaps at markedly reduced structural scales. Consequently, these systems provided a novel pathway for low-frequency vibration and noise control, presenting considerable theoretical interest and promising practical applicability. By virtue of their rationally engineered subwavelength microstructures, these materials exhibited extraordinary physical properties, such as negative effective mass density, negative bulk modulus, and forbidden bandgaps, which were not inherent in natural media. The exploitation of

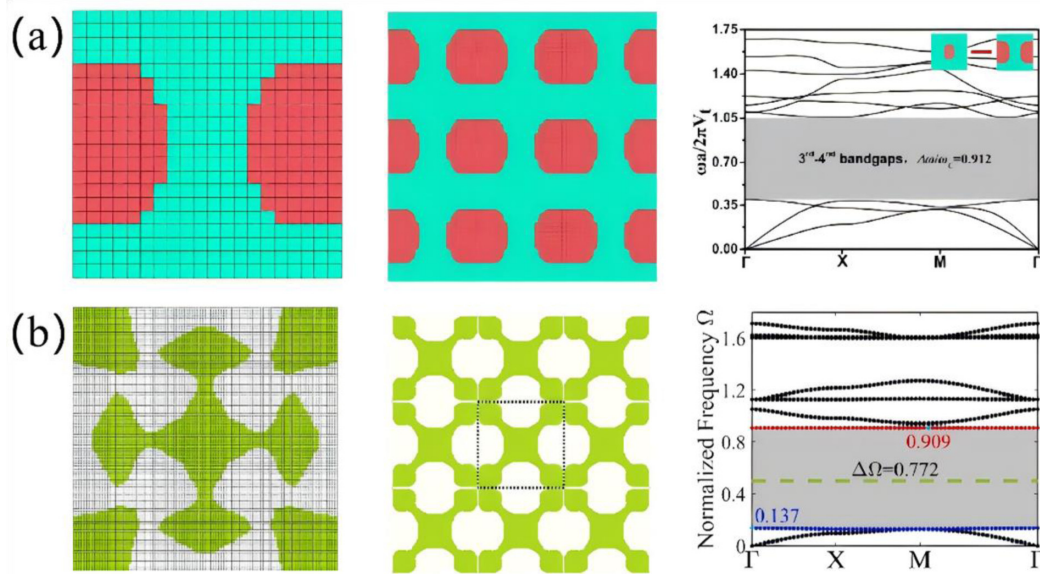


Fig. 2. (a) Ultrawide band gap design of PnCs based on topological optimization [20]. (b) Maximizing acoustic band gap in PnCs via topology optimization [21].

these properties facilitated precise manipulation of elastic and acoustic waves, including highly efficient absorption, reflection, and steering, thereby revealing considerable potential for addressing the long-standing challenge of low-frequency noise mitigation [12–17].

Bandgap is one of the most prominent characteristics of PnCs, also known as the attenuation zone. When the frequency of incident sound waves or elastic waves falls within this bandgap range, wave propagation through the PnCs rapidly attenuates, preventing further transmission through the crystal structure. This bandgap property offers novel approaches for vibration damping and noise reduction design. Furthermore, leveraging the bandgap properties of PnCs, novel acoustic functional devices such as filters [18] and waveguides [19] can be engineered. Localized resonator-based PnCs possess subwavelength tunability, enabling low-frequency bandgaps. However, their reliance on resonator resonance results in narrow bandgaps. In PnCs, bandgaps enable the control of a broader spectrum of waves, which translates into superior damping performance. Therefore, this paper will analyze recent research progress in vibration-damping and noise-reducing metamaterials from two perspectives: widening bandgap and tunable bandgap. It will also introduce their applications in specific engineering scenarios.

2 Widen the bandgap

Band Gap Widening refers to the process of increasing the band gap of a material through specific means or methods. The band gap denotes the energy difference between the bottom of the conduction band and the top of the valence band within a material, determining its electrical and optical properties. Band gap widening is typically pursued to enhance material performance for

specific application requirements. According to the latest academic research, recent methods for widening the band gap can be categorized into the following three types.

2.1 Phonon crystal cell design and optimization

The bandgap of PnCs could be significantly broadened through the strategic optimization of their unit cell architecture. A prominent approach involved employing topology optimization methods to design the unit cell, which enabled a broader bandgap [20,21]. As shown in Figure 2, the bandgap width was significantly enhanced by adjusting the geometric parameters and material distribution of cells.

The optimized unit cell structure was able to broaden the bandgap by enhancing interface reflection and reducing wave velocity, primarily through mechanisms such as introducing local resonances and lowering effective stiffness. Modifying the interconnection methods and effective stiffness between unit cells also represented a key strategy for optimizing bandgap characteristics. The incorporation of stiffness and connectivity constraints during the optimization process ensured interfacial continuity between adjacent unit cells while preventing the formation of isolated material domains, thereby achieving a balance between bandgap performance and structural manufacturability. Furthermore, the application of combined topological optimization approaches enabled the design of novel PnCs exhibiting broadband vibration suppression and negative refraction properties [22]. In 2000, Liu et al. pioneered a locally resonant PnCs. Its unit cell core was composed of a high-density lead sphere, which was coated with a soft silicone rubber layer and embedded in an epoxy resin matrix. This configuration generated strong local resonances that opened a sub-wavelength bandgap,

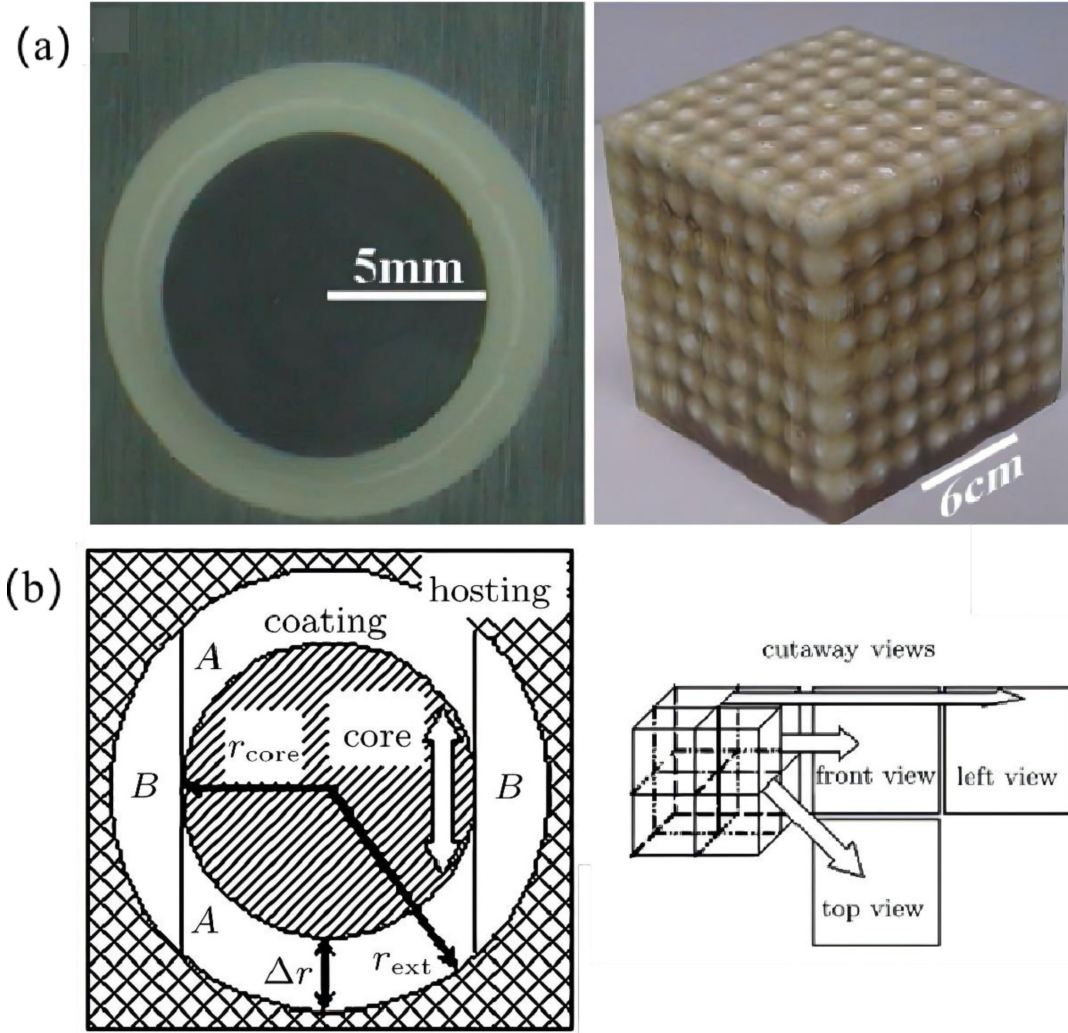


Fig. 3. (a) Locally resonant sonic materials [23]. (b) Accurate evaluation of lowest band gaps in ternary locally resonant [24].

enabling effective low-frequency noise and vibration attenuation [23]. The combination of a lead core and a silica coating behave like a spring-mass resonator with low stiffness and high mass, resulting in resonance modes at very low frequencies. The physical mechanism of BG is based on the coupling between the resonance modes of localized spherical units and the bulk modes within the crystal. In 2006, Wang et al. extended prior research by systematically investigating the lowest bandgap in ternary locally resonant PnCs (LR PCs). They introduced two analogous theoretical models the “mass-spring” model and the “mass-spring-mass” model to characterize the vibrational modes at the upper and lower edges of this bandgap [24]. The specific model constructed is shown in Figure 3. As shown in Figure 4, the start frequency f_1 (upper boundary frequency) and cutoff frequency f_2 (lower boundary frequency) of the bandgap were:

$$f_1 = \frac{1}{2\pi} \sqrt{\frac{k_1}{m_1}} \quad (1)$$

$$f_2 = \frac{1}{2\pi} \sqrt{\frac{k_1 \left(\frac{m_1}{m_2} + 1 \right)}{m_1}}. \quad (2)$$

In equations (1) and (2): k_1 is the equivalent vibration stiffness, primarily provided by the flexible covering layer on the outer surface of the scatterer; m_1 is the equivalent vibration mass of the scatterer; m_2 is the equivalent vibration mass of the substrate. These models, grounded in physical insights into bandgap mechanisms, offer a more intuitive explanation for the formation of localized resonant bandgaps. Based on the analysis of lattice vibration modes, the study proposed two simplified spring-mass models to predict the edge frequencies of the minimum locally resonant bandgap. The analogous model corresponding to the vibration mode is shown in Figure 4. These models evaluate the influence of structural and material parameters on these critical frequencies, with accuracy verified against the Lumped Mass method for system discretization. This approach enables a more precise theoretical framework for designing and applying PnCs.

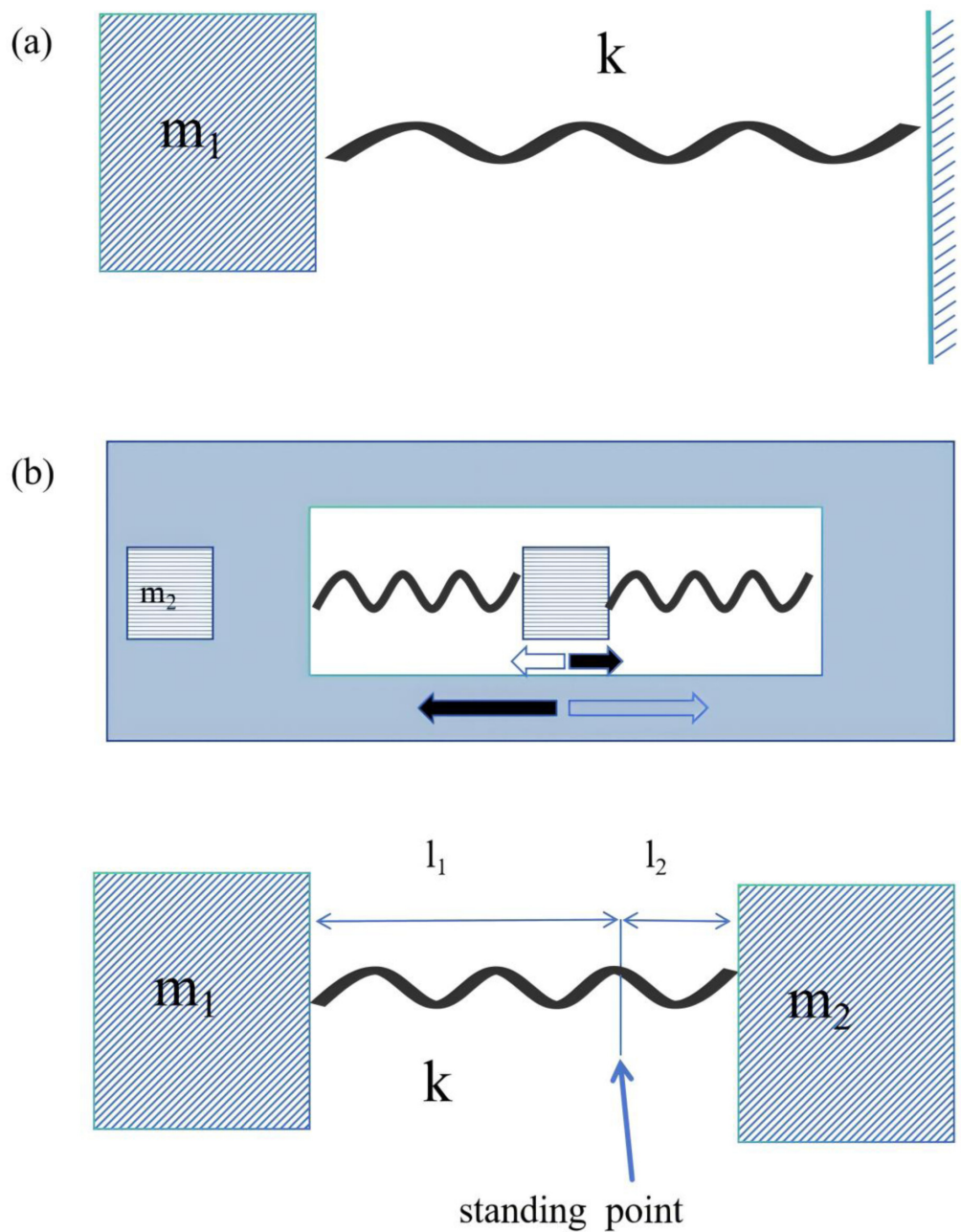


Fig. 4. Analogous models corresponding to vibration modes on the (a) lower and (b) upper edges of the lowest LR band gap.

The performance of PnCs is governed by the interplay between geometric architecture and material parameters, which collectively determine bandgap characteristics by modifying the effective mass ratio between scatterers and the matrix, thereby controlling the spectral position and width of bandgaps [25]. However, As shown in Figure 5a, geometry exerts a more profound influence by fundamentally altering vibrational modes; it modulates not only mass distribution but also equivalent stiffness [26]. A key mechanistic transition occurs when the dominant vibration mode shifts from translational to rotational motion: the effective vibrating mass transforms into rotational inertia,

and stiffness becomes torsional. This shift is dictated not solely by material properties but critically by geometric configuration.

Early research emphasized structural modifications to enhance performance. For instance, Naify et al. demonstrated that introducing multiple annular masses into membrane-type Locally Resonant Acoustic Metamaterials (LRAMs) could increase transmission loss (TL) peak bandwidth by up to 360% compared to a single central mass design [27]. By optimizing the number, distribution, and radius of these annular masses, multi-peak TL characteristics were achieved, expanding the >30 dB

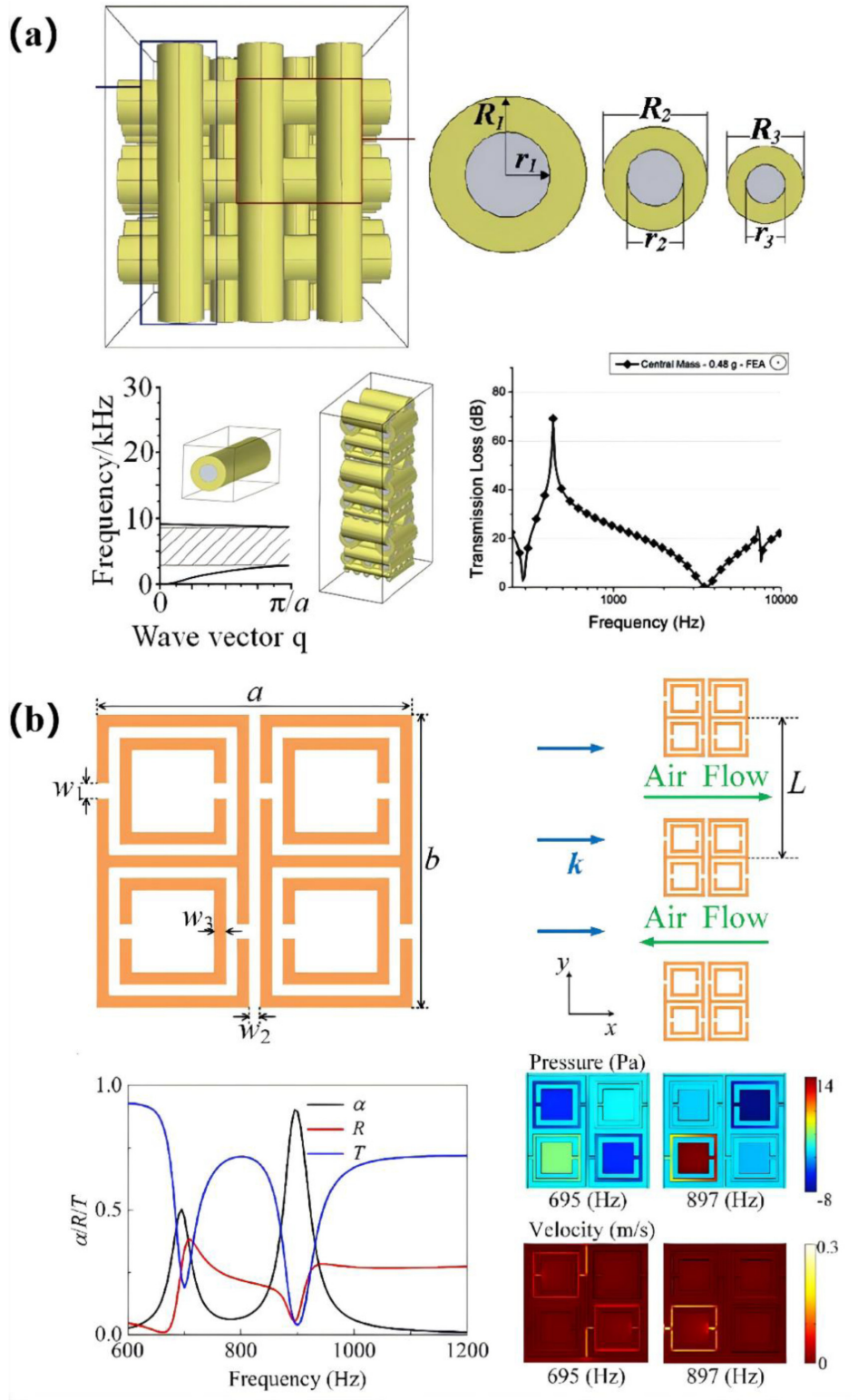


Fig. 5. (a) Locally resonant phononic woodpile: a wide band anomalous underwater acoustic absorbing material [26]. (b) Enhancing the acoustic absorption of vegetation with embedded periodic metamaterials [28].

bandwidth from 3.2 kHz to 5.5 kHz. This enhancement was validated experimentally and via finite element simulation. Crucially, this geometric innovation did more than resize components; it altered the physical mechanism by

promoting complex vibrational mode coupling and shifting energy into rotational degrees of freedom, thereby accessing a broader resonant spectrum. Similarly, the use of Genetic Algorithms (GA) to optimize complex metacells

that embedding resonators like split rings and Helmholtz resonators within a porous matrix with backing cavities, achieving near-perfect sound absorption over a broad frequency range of 1800–7000 Hz [27]. While still rooted in local resonance, this design relies on a distinct physical phenomenon, high-density overlapping resonances, strong inter-mode coupling, and spatial wave trapping within a subwavelength structure.

From a fundamental standpoint, these developments are unified by the overarching principle of local resonance [28]. However, they diverge in their operational physical mechanisms. The annular mass design primarily exploits inertial redistribution and modal hybridization (transitioning to rotational dynamics), whereas the GA-optimized metacell leverages dense resonance overlap and wave-energy localization. Engineering-wise, this represents a progression from parametric optimization (tuning mass/stiffness ratios within a fixed mechanism) to mechanistic innovation (using geometry to activate categorically different vibrational phenomena from single-mode translation to multi-mode rotation and collective resonance coupling), specific examples can be seen in Figure 5b. Thus, while connected by the foundational concept of resonance, each geometric strategy engages distinct physical mechanisms to achieve advanced acoustic control, highlighting that geometric design in PnCs and LRAMs is a powerful tool for selecting and enhancing specific wave-matter interaction pathways.

In 2016, Lin introduced a time-dependent adjoint-based shape optimization method for designing 2D acoustic metamaterials and PnCs [29]. They developed an acoustic wave solver using the SUPG time-domain scheme and parameterized surfaces via controlled meshing to efficiently handle complex geometries. For acoustic metamaterials, the approach optimized unit cell shapes to tailor effective properties, such as maximizing refractive index and minimizing impedance. For PnCs, it enhanced broadband noise attenuation by optimizing periodic structures. This method offers an efficient design tool particularly suited for broadband acoustic applications. In 2019, Xiang et al. [30] proposed a novel localized resonant PnCs structure featuring a “square-helix with in-band circles” configuration. The geometric parameters of the structure were optimized using a hybrid approach integrating Response Surface Methodology with the Finite Element Method. The results demonstrated a significant enhancement in bandgap characteristics: the initial frequency of the first bandgap was reduced to 191.8 Hz, while its bandwidth reached 115.9 Hz. The second bandgap achieved a bandwidth of 108.4 Hz, resulting in a total bandgap width of 222.9 Hz.

In addition, 3D printing technology have prompted a growing number of researchers to explore the design and fabrication of PnCs unit cells using single-material systems, moving away from traditional multi-material configurations. This shift in design paradigm accordingly requires that the classical mass-spring-mass system, traditionally implemented via contrasts in material properties, must now attain its target acoustic performance by means of rationally engineered geometries. At the same time, researchers have begun incorporating additional

considerations, such as structural strength, into the design process, thereby introducing a higher degree of tailorability into the cellular architectures. While this approach offers enhanced design flexibility and manufacturing convenience, it has also led to PnCs that predominantly exhibit bandgaps in higher frequency ranges [31–33]. In 2014, Li et al. proposed a paper two-dimensional phonon crystal [34]. The proposed structure comprises periodically arranged cross-shaped grooves (Fig. 6a). By employing the finite element method in conjunction with Bloch's theorem, the bandgap characteristics were systematically investigated. Results demonstrate that the structure possesses a pronounced bandgap within the low-frequency region. Moreover, both the central frequency and bandwidth of this bandgap can be effectively tuned across a remarkably broad range by varying the length and width of the grooves. In 2015, Wang et al. [35] investigated a novel two-dimensional PnCs whose unit cells consisted of periodic S-shaped grooves embedded in an air matrix (Fig. 6b). Utilizing the finite element method in conjunction with Bloch's theorem, the dispersion relations, acoustic pressure fields, and transmission spectra were calculated. The results identified multiple complete bandgaps and broader directional bandgaps in the low-frequency range, with the lowest bandgap extending from 1647 to 2124 Hz.

Compared to Jerusalem cross-shaped groove structures, this S-groove design demonstrated bandgaps at lower frequencies, greater bandwidth and heightened sensitivity to variations in geometric parameters. Analysis of the acoustic pressure field indicated that bandgap formation originates predominantly from resonant modes localized within the S-shaped groove cavity. These resonant modes showed strong dependence on geometric parameters, including the horizontal length (l) and vertical lengths (m and n). Specifically, increasing the horizontal length (l) was found to shift the bandgap to lower frequencies while slightly increasing its width; increasing the vertical length (n) lowered the bandgap frequency but reduced its width; and variations in the vertical length (m) had more complex effects on both the bandgap position and width. The S-groove structure thus allows flexible tuning of low-frequency bandgaps through geometric optimization, providing valuable insights for the design of PnCs, particularly for engineering applications such as low-frequency noise control and acoustic filtering.

In 2016, Wang investigated a novel two-dimensional PnCs whose unit cell consists of periodically arranged spindle-shaped lead inclusions embedded in a rubber matrix (Fig. 6c) [36]. Using the finite element method combined with Bloch's theorem, the dispersion relations, transmission spectra, and displacement fields of this structure were calculated. The results reveal a significant complete bandgap within the low-frequency range of 163–222 Hz, corresponding to a relative width of 15%, which demonstrates its suitability for low-frequency noise and vibration control. Furthermore, analysis of the effective mass density showed it becomes negative within the bandgap, leading to wave velocity complexification and enabling exponential wave attenuation. The study also investigated how geometric parameters of the inclusions such as center height, side height and width influence the

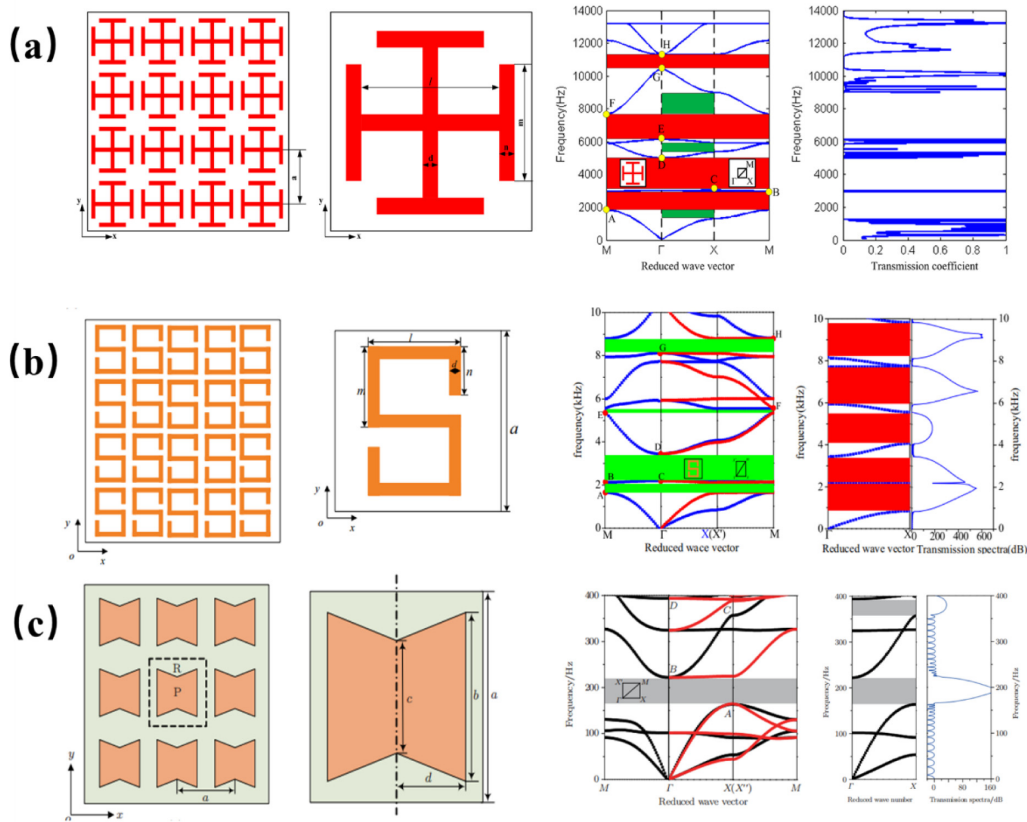


Fig. 6. Band structures in two-dimensional PnCs: (a) periodic Jerusalem cross slot [34], (b) periodic s-shaped slot [35], (c) spindle-shaped inclusions [36].

bandgap. These parameters were found to modulate both the position and width of the bandgap across a broad frequency range. Through innovative unit cell design and geometric parameter optimization, the spindle-shaped inclusion phonon crystal demonstrated superior performance in low-frequency bandgap tuning.

The bandgap design of traditional PnCs has traditionally relied on contrasts in material properties, as exemplified by the classical “mass-spring-mass” model. However, with the increasing maturity of 3D printing technology, single-material designs have progressively emerged as a dominant approach. Through deliberate geometric design such as the introduction of local resonances, periodic arrangements, or specific structural configurations, bandgaps can now be effectively realized without dependence on material heterogeneity. In 2021, An et al. fabricated a three-dimensional, single-phase monolithic plate lattice structure incorporating a chiral microstructure (Fig. 7a) [37]. Chirality was achieved by etching grooves onto the surfaces of units within a high-load-bearing plate lattice. When elastic waves propagate through the lattice, multiple vibration modes induced by the chiral surfaces dissipate energy, resulting in broadband vibration suppression. Adjusting parameters of the chiral microstructure such as the rib position x , which modifies the panel's chirality, thereby tuning the bandgap characteristics and spectral locations. The addition of circular mass blocks to the central region alters the microstructure's mass distribution

for bandgap adjustment. Specifically, adding mass blocks to panels perpendicular to the wave propagation direction shifts the first bandgap to lower frequencies as the block thickness increases. Incorporating mass blocks into panels along the beam extension direction extends the second bandgap, lowers the cutoff frequency of the third bandgap, and induces an overlapping effect with the third bandgap, thereby generating a broader bandgap while also shifting the first bandgap's lower boundary toward lower frequencies (Fig. 7b) [38,39]. Experimental results demonstrate significant vibration isolation across multiple frequency bands, with maximum attenuation levels of 42 dB, 52 dB and 48 dB, respectively. The sound absorption coefficient exceeds 80% within the 1700–5100 Hz range, reaching 99.18% at 2500 Hz. Quasi-static compression tests confirm the MPL structure's excellent load-bearing capacity, with compressive strengths of 0.25 MPa for single-cell and 1.41 MPa for multi-cell configurations.

2.2 Introduction of the mechanism for inertial volume expansion

Classical mechanical systems are fundamentally composed of three elemental components: mass, spring, and damper. A significant conceptual advance was made by Smith [40], who introduced the “inertia element” as a novel mechanical component. In this framework, each inertia element possesses two ports connecting adjacent nodes, with the

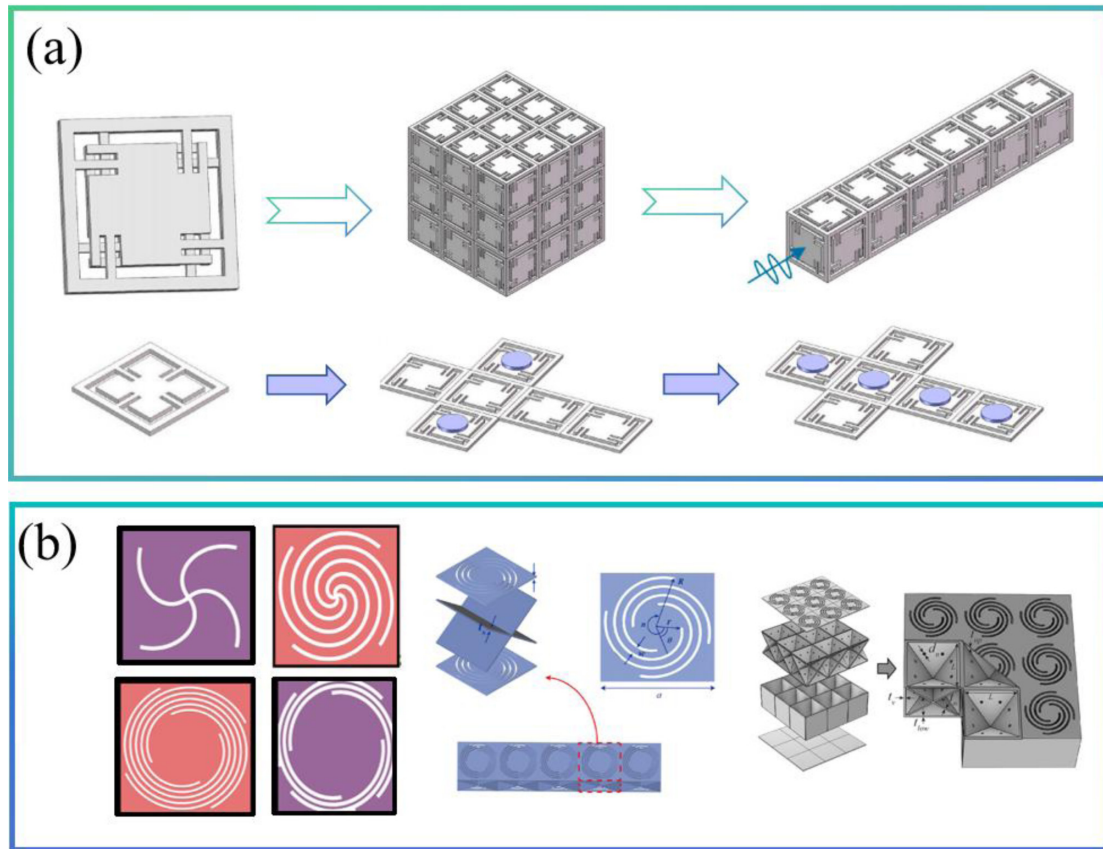


Fig. 7. (a) Three-dimensional chiral meta-plate lattice structures for broad band vibration suppression and sound absorption [37]. (b) Deep-subwavelength lightweight metastructures for low-frequency vibration isolation [38, 39].

applied force being proportional to the relative acceleration between the nodes. Smith emphasized the limitations of conventional mass elements in dynamic networks and formally defined the inertia element as a two-port mechanical component governed by relative acceleration. His work further explored applications in vibration absorption, vehicle suspension design, and mass simulation, demonstrating the unique advantages of such elements in mechanical network synthesis.

In recent years, metamaterials research has witnessed considerable growth, with numerous studies exploring innovative structural mechanisms. Among these, the inertia amplification mechanism has attracted particular interest. Accumulated evidence indicates that, under specific design conditions, the incorporation of inertia amplification can significantly widen the bandgap of metamaterials. Bandgaps effectively block elastic waves or vibrations within certain frequency ranges; broadening them enhances wave suppression over a wider spectrum, thereby improving performance in vibration isolation, noise reduction, and acoustic wave control. Through sophisticated structural design or optimized inertial distribution, the inertia amplification mechanism strengthens the suppression of elastic waves, offering a powerful strategy for performance enhancement in metamaterial-based applications. In 2007, Yilmaz introduced a novel approach for inducing phononic bandgaps in periodic media via an inertial amplification mechanism (Fig. 8a)

[41], designing a two-dimensional mass-spring lattice as a representative model. The lattice consisted of triangular trusses formed by structural nodes of mass m_m , with smaller nodal masses m_a connected via springs of stiffness ka to constitute the amplification unit. When structural nodes experienced relative displacement, amplified displacements were generated at the smaller nodes, achieving the inertial amplification effect. The study demonstrated that this mechanism could produce broad bandgaps in the low-frequency regime. Furthermore, it enabled the formation of wide low-frequency bandgaps without relying on high mass fractions of embedded amplifiers. This strategy provided valuable insights for the design of metamaterials, particularly for applications in vibration isolation, noise reduction, and acoustic wave control. In 2012, Yilmaz further investigated phonon bandgap properties induced by inertial amplification in finite periodic structures (Fig. 8b) [42]. A two-dimensional finite periodic lattice model was developed, incorporating an inertial amplification mechanism consisting of large nodes (with spring stiffness k_k and mass m_m) and small nodes (with spring stiffness k_a and mass m_a). When structural nodes experienced relative displacement, the small nodal masses (m_a) underwent amplified displacement, thereby realizing inertial amplification. The study demonstrated that this mechanism could generate broad and deep bandgaps in the low-frequency range, with performance remaining robust against variations in boundary conditions, excitation

relationships, it amplifies inertial forces to enhance wave attenuation performance. Multi-stage XIA mechanisms, assembled in a vertical configuration, further improve wave control capabilities. Additionally, XIA mechanisms induce coupling effects between transverse and longitudinal waves, generating new bandgaps. Parametric studies reveal that variations in the inertia amplification angle, amplification ratio, and span significantly influence bandgap characteristics.

Taniker designed a novel inertia amplification mechanism topology [45]. By incorporating inertia amplification units with flexible hinges into the periodic structure, the vibration cutoff band was significantly broadened. The structure is composed of rigid rods and mass blocks, which amplify inertial forces through specific geometric configurations and kinematic relationships. In contrast to conventional Bragg scattering and local resonance methods, this design achieves an exceptionally broad low-frequency bandgap. The bandgap was further widened by optimizing geometric parameters of the unit-cell mechanism, such as the length and thickness of the beam segments. Numerical and experimental results demonstrate that the optimized structure exhibits a broad low-frequency bandgap spanning from 7.8 to 303.6 Hz, with a normalized bandwidth of 6.10, significantly surpassing conventional approaches. This ultra-wideband performance is achieved by enhancing the effective inertia, constraining the sliding motion of the beams, and utilizing the bending mode of the constrained beam-spring system. As a result, the upper limit of the first bandgap is determined by the second vibrational mode. Moreover, by altering material properties for instance, substituting steel with aluminum for the thick beams and the frequency range of the bandgap can be adjusted while largely preserving the normalized bandwidth. Li et al. [46] periodically incorporated an inertia amplification mechanism onto Euler-Bernoulli beams, thereby enhancing the structure's effective inertia and generating an ultra-wide cutoff band in the low-frequency range. Compared to local resonance methods, this approach achieves a broader low-frequency bandgap without sacrificing stiffness or increasing overall mass. Kalderon et al. (Fig. 8e) [47], designed a more concise dynamic directional inertia amplifier in 2024, fixing the vibrating mass to the free end of a hinged rod to constrain its trajectory. This transforms its vibration mode from translational motion to rotational motion about an axis, thereby achieving inertia amplification. The team applied this to phonon crystal design, demonstrating that incorporating the inertia amplification mechanism enables adjustment of the bandgap position and broadening of the bandwidth.

2.3 Combining multiple narrowband connections into a broadband connection

During the last century, techniques for synthesizing broadband signals from multiple narrowband components were implemented in electronic circuitry. These methods utilized directly coupled multivibrator configurations in

conjunction with antenna parameter optimization to achieve significant bandwidth expansion [48,49]. Researchers studying longitudinal wave bandgaps in elastic rods based on multi-degree-of-freedom resonators developed a combined approach employing spectral element methods and Bloch's theorem to compute complex band structures. This methodology enabled systematic investigation of resonator parameters' effects on bandgap characteristics. In parallel studies of localized resonant beams containing multiple periodic resonator arrays, the plane-wave expansion method was extended to analyze bending wave propagation. Results demonstrated that multi-array configurations achieved substantially broader bandgaps across wider frequency ranges compared to single-array beams. Furthermore, cell element design and optimization strategies included techniques for merging adjacent bandgaps to enhance overall attenuation performance.

D'Alessandro proposed a method to broaden the bandgap of three-dimensional elastic periodic structures through a modal separation strategy [50] (Fig. 9a). Researchers achieved an ultra-wide bandgap by decoupling low-frequency global modes from high-frequency local modes, creating a sharp modal mass disparity at the bandgap boundary. A specialized unit cell with rigid spheres and flexible connectors utilized beam bending stiffness rather than axial stiffness to lower the bandgap's starting frequency and increase its bandwidth. Parameters such as sphere mass and beam stiffness were optimized using a 1D spring-mass model for precise bandgap control. A 3×3×3 nylon prototype was fabricated, and transmission tests validated numerical simulations, with damping effects incorporated for accuracy. Collectively, these strategies enabled the 3D elastic periodic structure to function as a low-pass mechanical filter in the audible range, significantly broadening the bandgap. Li et al. [51] undertook the optimization of multiresonant piezoelectric metamaterial plates using a genetic algorithm, an approach centered on the phenomenon of bandgap coalescence to achieve significant bandgap broadening (Fig. 9b). They employed effective medium theory to derive the equivalent bending stiffness and dispersion relations of the metamaterial plate, establishing criteria for identifying bandgap ranges both with and without damping. Using a genetic algorithm as the optimization framework, they optimized the distribution of poles and zeros under multipole conditions, with the objective of maximizing the frequency range of the widest bandgap. By adjusting the magnitude and position of poles along with damping parameters, multiple independent bandgaps were merged, resulting in significant bandgap broadening. The study revealed that while only marginal widening was achievable in the absence of damping, introducing damping enabled a remarkable bandgap expansion of over 200% through this merging mechanism. Numerical simulations validated the bandgap widening effect, confirming the efficacy of the approach for enhancing piezoelectric metamaterials. This method improves vibration suppression at target frequencies and allows active tuning of the suppression frequency range.

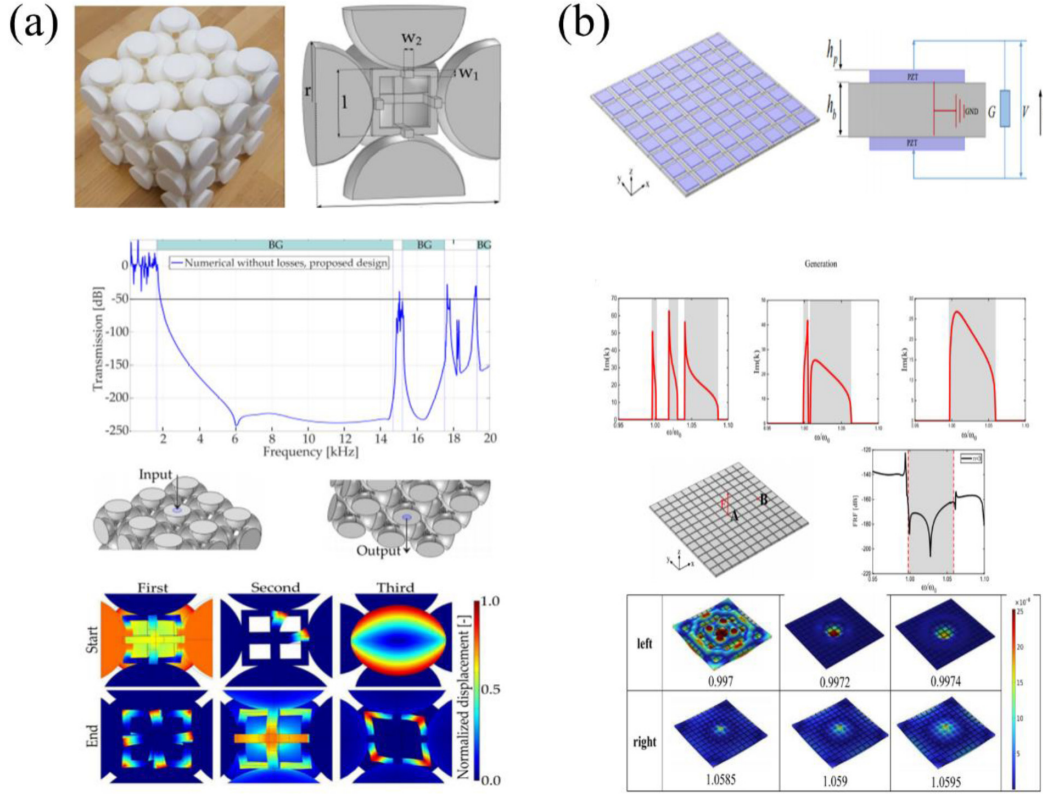


Fig. 9. (a) Mechanical low-frequency filter via modes separation in 3D periodic structures [50]. (b) Broadening bandgaps in a multi-resonant piezoelectric metamaterial plate via bandgap merging phenomena [51].

3 Adjustable bandgap

The bandgap of traditional metamaterials and the frequency range within which they exert their effect is largely fixed post-fabrication, making it difficult to adapt to complex and variable real-world conditions. To address this limitation, researchers have explored tunable bandgap metamaterials. Montgomery et al. [52] proposed a method employing functional materials, such as magnetic materials and magnetorheological materials, and shape memory materials, whose properties change in response to external stimuli, to dynamically reconfigure unit cell geometry and thereby tune the bandgap. Magnetic materials undergo rearrangement of internal magnetic domains under magnetic fields, altering their electromagnetic and mechanical properties. These changes can be transferred to the metamaterial's unit cell structure, modifying its geometric configuration and adjusting the bandgap. Alicia et al. [53] demonstrated an adjustable acoustic metamaterial cell using a magnetic membrane for tunable resonance in the sub-100Hz regime. Fabricated via stereolithography, the cells mechanical and acoustic properties were characterized using laser Doppler vibrometry and electret microphone testing. The results showed that the cell exhibited adjustable resonance frequencies ranging from 88.73 to 86.63Hz under varying applied magnetic fields. Magnetorheological materials exhibit significant rheological changes in magnetic fields, transitioning from low-viscosity fluid-like states to

high-viscosity or solid-like states. This property shift can be harnessed to alter the position or shape of movable components within the metamaterial, enabling bandgap tuning. BAYAT et al. [54] employed an energy-based approach to theoretically analyses wave propagation characteristics in porous hyperplastic magnetorheological elastomers subjected to external magnetic fields. Their model incorporated large deformations induced by external forces. Under first-order buckling deformation modes, the original circular holes transformed into elliptical ones, with the major axes of adjacent elliptical holes oriented perpendicular to each other. Consequently, the structure's minimal periodic unit expanded to twice its original size. Finite element model calculations indicate that the overall bandgap increases and broadens with magnetic field enhancement, attributable to the rise in magnetoelastic modulus. Shape memory materials possess a unique shape memory effect, allowing reversible transitions between multiple shapes when stimulated by external factors such as temperature. Montgomery et al. [52] integrated these materials into metamaterial structures, enabling precise, temperature-controlled shape transformations, dynamically adjusting structural geometry to modulate the bandgap. These strategies provide novel pathways for the development of tunable bandgap metamaterials, enhancing their potential for practical applications across diverse fields.

Wang et al. [55–57] conducted in-depth research into the extraordinary physical properties of PnCs (Fig. 10a), designing tunable broadband helical metasurfaces and

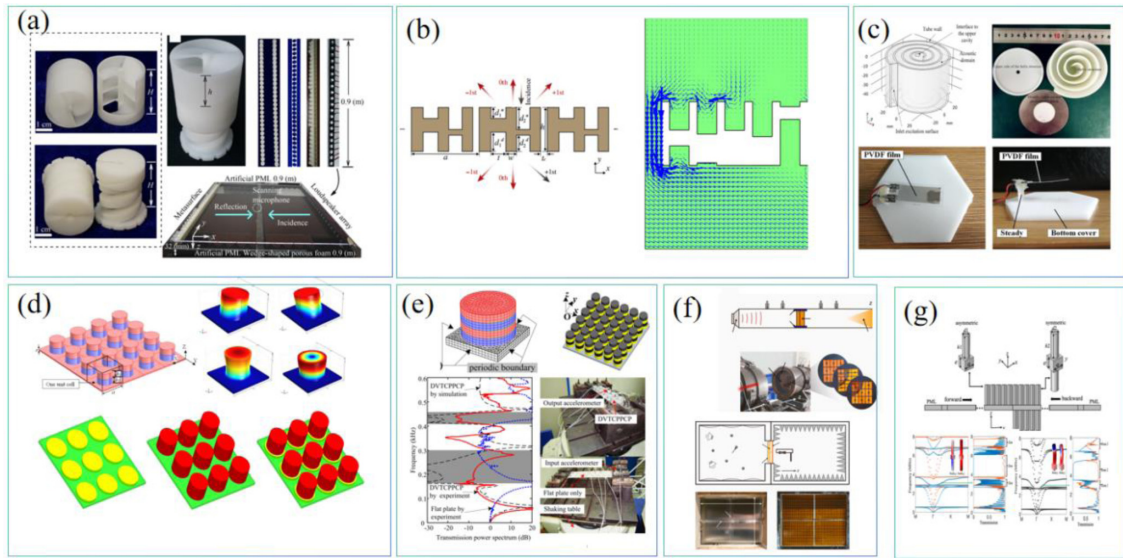


Fig. 10. (a) Tunable broadband reflective acoustic metasurface [55]. (b) Highly efficient acoustic metagrating with strongly coupled surface grooves [62]. (c) Helix structure for low frequency acoustic energy harvesting [64]. (d) A sonic band gap based on the locally resonant phononic plates with stubs [69,71]. (e) Vibration band gaps in double-vibrator pillared PnCs plate [80]. (f) Laminated plate-type acoustic metamaterials with Willis coupling effects for broadband low-frequency sound insulation [74]. (g) Broadband asymmetric propagation in pillared meta-plates [78].

reconfigurable curved acoustic metasurfaces for acoustic cloaking and acoustic illusions. This work provides viable pathways for achieving broadband tunability in multi-functional acoustic metasurfaces. Furthermore, the authors proposed a curved-wave cloak with broadband characteristics, establishing a theoretical foundation for the design and fabrication of acoustic cloaking devices. In a complementary line of research, Wu et al. [58–60] conducted extensive studies in the areas of low-frequency vibration damping, noise reduction, and filter design, developing a micro-perforated plate sound-absorbing structure that shows significant potential for practical noise control applications. They also proposed an acoustic valley Hall system to achieve multi-band selective acoustic valley transport, providing a theoretical foundation for potential engineering applications such as multi-band communications and filters. Hou conducted in-depth investigations into defect state characteristics, metasurfaces, and piezoelectric systems, advancing fundamental research for the broad application of PnCs (Fig. 10b) [61–63]. In expanding the functional capabilities of PnCs, he designed more practical structures by incorporating functional materials and coupling diverse physical fields, enabling applications such as active bandgap control and vibration energy harvesting (Fig. 10c) [64–66]. With the continuous advancement in PnCs research, plate-like structures have demonstrated superior practical applicability compared to one-dimensional beam or three-dimensional bulk structures in terms of active bandgap control and vibrational energy harvesting. This advantage stems from the predominant propagation of elastic waves as surface waves within plate structures, which effectively reduces energy dissipation in the medium. Concurrently, researchers have observed that PnCs plates can broaden the bandgap frequency range, enabling their extensive

application in low-frequency vibration damping and noise reduction. Initial studies on PnCs plate structures primarily focused on two-component systems, where researchers modulated the bandgap by altering the geometric configurations of both the substrate and the scattering layer, leveraging the material contrast between them (Fig. 10d) [67–69]. Oudich et al. used the finite element method to analyze the acoustic behavior of single- or double-layer PnCs structures embedded in thin plates. Numerical results showed that extremely low-frequency bandgaps can be generated through local resonance mechanisms, with the bandwidth exhibiting a direct dependence on both the height and cross-sectional area of the phononic units. In a related study, Chen et al. [70, 71] examined a two-dimensional, two-component locally resonant plate with single-sided stubs (Fig. 10d), demonstrating that the bandgap arises primarily from local resonance coupling between the base plate and the mass blocks, resulting in bandgaps within the low-frequency regime. Assouar et al. demonstrated that structural modifications such as converting a unilateral configuration to a bilateral one, or transforming a single-sided two-component system into a three- or multi-component system through superposition—systematically influenced the bandgap frequency. These transformations altered the dynamic coupling within the PnCs (Fig. 10e), leading to predictable shifts in the bandgap characteristics without fundamentally changing the underlying physical mechanism [72].

Gu et al. [73–75] designed a localized resonator plate structure with an interlayer (Fig. 10f). When elastic waves propagate through the plate structure, they interact with the localized resonator structure. The increased propagation path through the interlayer structure leads to greater energy dissipation, thereby reducing the energy and

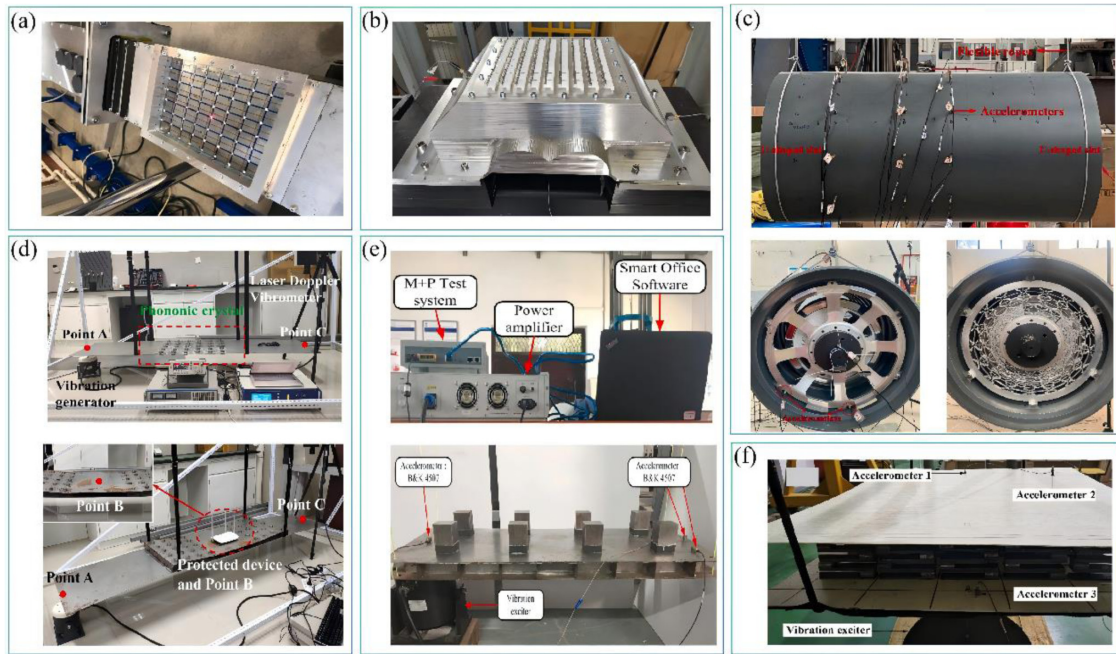


Fig. 11. (a) The use of locally resonant metamaterials to reduce flow-induced noise and vibration [81]. (b) Lightweight gearbox housing [82]. (c) Underwater vehicle power cabin [84]. (d) Power systems on a ship [85]. (e) Low frequency ship vibration isolation [86]. (f) Sandwich plate structure periodically attached by S-shaped oscillators for low frequency ship vibration isolation [87].

achieving the effect of energy attenuation. Zhou et al. [76, 77] discovered that by periodically embedding multilayer scatterers into a matrix, the bandgap of localized resonant PnCs could be extended across multiple frequency ranges. Jin et al. [78] proposed a plate-like asymmetric structure composed of symmetrically and asymmetrically distributed pillars. Within the asymmetric region (Fig. 10g), elastic waves exhibited energy attenuation exceeding 200 dB, enabling a broadband gap. Zhang et al. [79] proposed a symmetric power-law columnar PnCs structure, discovering that it exhibits three complete bending wave bandgaps, with the second bandgap width of 1639 Hz.

4 Applications in vibration and noise reduction

In recent years, researchers have increasingly explored the application of vibration-damping and noise-reducing metamaterials across various engineering fields, including building structures, automotive systems, high-speed railways, and marine vessels. Particularly in marine and naval engineering, PnCs have garnered significant attention and achieved notable breakthroughs in vibration and noise control, contributing to the development of a quieter and more stable marine environment (Fig. 11a and 11b) [81, 82]. In the field of ocean engineering, Vasconcelos et al. designed a metamaterial-based joint for offshore monopiles to shield low-frequency vibration noise during pile driving [83]. Zuo et al. developed a star-shaped metamaterial array for broadband transient acoustic vibration predictive control of the power compartment in underwater vehicles (Fig. 11c) [84]. In the field of marine engineering, Ruan et al. applied a spiral beam PnCs to a ship propulsion

system, achieving a low-frequency bandgap from 15 to 45 Hz (Fig. 11d) [85]. Chen et al. employed a sandwich panel structure featuring “rubber-pad-steel-pillar” resonators, achieving a vibration suppression band of 78 to 142 Hz (Fig. 11e) [86]. Shen et al. similarly employed a sandwich panel structure, featuring an S-shaped resonator resembling a spring between the upper and lower panels, achieving bending wave bandgaps from 45 to 78 Hz and 145 to 355 Hz (Fig. 11f) [87]. The structure designed by Ruan et al. achieved a lower frequency band for the bandgap, while the sandwich panel structure with “rubber pad-steel pedestal” resonators designed by Chen et al. [52] demonstrated superior strength characteristics [85].

5 Conclusions

According to classical acoustic theory, the vibration and noise reduction performance of a structure or system is primarily determined by its impedance characteristics, which in turn are governed by both the material properties and geometric parameters of the system. Metamaterials achieve enhanced vibration and noise reduction by deliberately designing the geometry and material composition of the structure to tailor its equivalent material parameters, thereby meeting specific acoustic impedance requirements. Since the introduction of locally resonant PnCs. Researchers have focused on expanding their bandgap properties and enhancing structural functionality. On one hand, through the design of novel unit cells and non-periodic metamaterial structures, combined with techniques such as inertial amplification and multi-narrowband merging, efforts have been made to overcome the inherent limitation of narrow bandgaps. On the other hand, developments have

progressively extended toward tunable bandgaps and multifunctional integration. Several key aspects in recent research deserve particular attention: The inertial amplification mechanism not only effectively broadens bandgaps but also contributes to lightweight structural design; Spiral structures and membrane-type acoustic metamaterials exhibit excellent performance in exciting low-frequency resonances; however, their mechanical strength is often insufficient. Balancing acoustic performance with structural strength requirements remains a critical challenge in current structural design; In the design of PnCs unit cells, the modal separation strategy provides an effective approach for bandgap regulation. Magnetic shape memory metamaterials integrate both magnetic and thermal tuning mechanisms, allowing flexible adjustment of bandgap position and width, and even enabling the “switching-off” of bandgaps. This switching characteristic offers new possibilities for controlling the distribution of vibrational energy along transmission paths. For example, by placing dynamic vibration absorbers on both sides of a bandgap switch, balanced energy dissipation between the two can be achieved when the bandgap is closed, whereas one absorber may dominate energy dissipation when the bandgap is open.

Furthermore, recent years have seen preliminary attempts to apply vibration and noise reduction metamaterials in practical engineering. Acoustic metamaterials, with their sub-wavelength modulation capability, are particularly suitable for low-frequency vibration suppression. They show great potential in controlling vibration and noise from power machinery such as ship engines, thereby improving vessel comfort and structural durability. Although current engineering applications are still in the early stages, there remains considerable scope for future development in this field.

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Conflicts of interest

The authors declare no conflict of interest.

Author contribution statement

Yinglong Ma performed methodology, investigation, data curation, and wrote the original draft and performed formal analysis, investigation and data curation. Zongxiang Wang performed methodology, validation, formal analysis and investigation. Yuan yuan performed data curation, formal analysis, and investigation.

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